

RESEARCH CENTRE

**Inria Saclay Centre  
at Institut Polytechnique de  
Paris**

IN PARTNERSHIP WITH:

EDF R&D, Institut Polytechnique de Paris

2023

ACTIVITY REPORT

Project-Team

IDEFIX

**Inversion of Differential Equations For  
Imaging and physIX**

**DOMAIN**

**Applied Mathematics, Computation and  
Simulation**

**THEME**

**Numerical schemes and simulations**

*Inria*

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## Project-Team IDEFIX

*Creation of the Project-Team: 2021 August 01*

### Keywords

#### Computer sciences and digital sciences

- A6.1. – Methods in mathematical modeling
  - A6.1.1. – Continuous Modeling (PDE, ODE)
  - A6.1.4. – Multiscale modeling
  - A6.2.1. – Numerical analysis of PDE and ODE
  - A6.2.6. – Optimization
  - A6.2.7. – High performance computing
  - A6.3.1. – Inverse problems
  - A6.5.4. – Waves
- A9.2. – Machine learning

#### Other research topics and application domains

- B1.2.3. – Computational neurosciences
- B2.6.1. – Brain imaging
- B3.3.1. – Earth and subsoil
- B3.3.2. – Water: sea & ocean, lake & river

# 1 Team members, visitors, external collaborators

## Research Scientists

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## Post-Doctoral Fellows

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## PhD Students

- Chengran Fang [INRIA, until Jan 2023]
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## Technical Staff

- Alex Mc Sweeney-Davis [INRIA, Engineer, from Nov 2023]

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## Visiting Scientist

- Slim Chaabane [Sfax University]

## External Collaborators

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- Frederic Taillarde [EDE, HDR]
- Denis Vautrin [EDF]

## 2 Overall objectives

Inverse problems are encountered in many real life applications and the ones we are interested in are those that can be formulated as parameter identifications in a PDE (system) modeling physical phenomena, primarily wave propagation and diffusion. As opposed to determining the solution of the forward model, identifying the parameters from measurements of this solution usually leads to an unstable and non linear problem that may be not uniquely solvable. A standard method to formulate this inverse problem is to consider it as a minimization of a cost functional that measures data fidelity. The solution to the latter is computationally much more costly than solving the PDE and may even not be realistic for number of applications that require real time answers or for very large scale problems. These considerations motivated the research guidelines items exposed above and that we shall develop further in the following.

At EDF, the need for algorithms to solve inverse problems is present in numerous applications (see Section 4 for instance). The team Signal, Image and Learning at EDF R&D, PRISME has developed solutions mainly based on signal processing methods that do not require fine modeling of the physical phenomena (describing the experiment). This enables fast simplified responses that can usually be satisfactory. The complexification of the measuring devices and environments appealed for more precise assessment of the experiments and therefore for more reliable/precise inversion methods. This was the motivation behind the intense collaborations between EDF R&D and the DEFI project team that lead to six co-supervised PhD thesis and one PostDoc on various themes (Eddy current imaging for pipes, data assimilation for primary cooling loops, sampling methods for concrete like materials, multi-element eddy current 3-D probes, qualitative inversion methods and spectral signatures for ultrasound applications). The joint team aims at pursuing this collaborative effort that has been beneficial to both partners, motivating at the same time fundamental research to establish solid theoretical foundations of promising inversion methods and (non trivial) adaptations of established methods to solve applications of interest for EDF.

## 3 Research program

Let us describe the outline of the main challenges that we would like to address for solutions to inverse problems, taking as a guideline the example of non destructive testing which is central for EDF applications. A typical experiment would be to probe some defects inside a given structure by sending waves that can propagate inside the domain of interest. The response of the media is recorded by some receivers and forms the data of the inverse problem. We can distinguish two types of inverse problems. In the first type, referred to by “imaging”, one is interested by only the location and/or the shape of the defect/inclusion. In the second one, referred to by “identification”, one is interested in getting information on the defect physical properties. Both problems (imaging and identification) are non linear and ill-posed (lack of stability with respect to measurements errors if some careful constrains are not added). Moreover, the unique determination of the geometry and/or the coefficients is not guaranteed in general if sufficient measurements are not available. As an example, in the case of anisotropic inclusions, one can show that an appropriate set of data uniquely determine the geometry but not the material properties. These theoretical considerations are usually difficult to address and are not only important in understanding the mathematical properties of the inverse problem, but also guide the choice of appropriate numerical strategies (which information can be stably reconstructed) and also the design of appropriate regularization techniques and improve the measurement techniques. Moreover, uniqueness proofs can be constructive proofs, i.e. they implicitly contain a numerical algorithm to solve the inverse problem, hence their importance for practical applications. The sampling methods introduced below are one example of such algorithms. As a complementary notion to identifiability is the notion of invisibility. The latter topic has attracted a large attention in the inverse problem community due in particular to the recent and rapid development of metamaterials that made plausible the design of cloaking devices based on transformation optics. However, these transformations require the use of non dissipative materials exhibiting singular physical coefficients taking infinite values, which is indeed not realistic. This motivated us to consider a weaker notion of invisibility where one would like to achieve invisibility for only a finite set of measurements and frequencies. This objective is less ambitious and consequently, it is more easily

achievable. On the other hand, it is pertinent from a practical point of view because one always has a finite number of sensors and very often, one has only access to a small number of measurements. In addition to theoretically investigate this issue for some idealized models, we would like to fructify our findings for non destructive testings in waveguides.

An important part of our research activity is dedicated to numerical methods applied to the first type of inverse problems, where only the geometrical information is sought. In its general setting the inverse problem is very challenging and no method can provide a universal satisfactory solution to it (regarding the balance cost-precision-stability). This is why in the majority of the practically employed algorithms, some simplification of the underlying mathematical model is used, according to the specific configuration of the imaging experiment. The most popular ones are geometric optics (the Kirchhoff approximation) for high frequencies and weak scattering (the Born approximation) for small contrasts or small obstacles. They actually give full satisfaction for a wide range of applications as attested by the large success of existing imaging devices (radar, sonar, ultrasound, X-ray tomography, etc.), that rely on one of these approximations.

Generally speaking, the used simplifications result in a linearization of the inverse problem and therefore are usually valid only if the latter is weakly non-linear. The development of these simplified models and the improvement of their efficiency is still a very active research area. With that perspective we are particularly interested in deriving and studying higher order asymptotic models associated with small geometrical parameters such as small obstacles, thin coatings, periodic media, ... Higher order models usually introduce some non linearity in the inverse problem, but are in principle easier to handle from the numerical point of view than in the case of the exact model. Asymptotic analysis is also a corner stone in our methodology to prove invisibility for finite number of measurements.

A major research axis is dedicated to algorithms that avoid the use of such approximations and that are efficient where classical approaches may fail: i.e. roughly speaking when the non linearity of the inverse problem is sufficiently strong. This type of configuration is motivated by the applications mentioned below, and occurs as soon as the geometry of the unknown media generates non negligible multiple scattering effects (multiply-connected and closely spaces obstacles) or when the used frequency is in the so-called resonant region (wave-length comparable to the size of the sought medium). It is therefore much more difficult to deal with and requires different approaches such as sampling methods. The sampling methods are fast imaging solvers adapted to multi-static data (multiple receiver-transmitter pairs). Even if they do not use any linearization of the forward model, they rely on computing the solutions to a set of linear problems of small size, that can be performed in a completely parallel procedure. Our team is among the leading international groups in the developments of these techniques. We are one of the main contributors in recent advancements in this field and actively acting in its dissemination among the academic and industrial communities. We shall pursue our efforts in developing and promoting these techniques. A closely related subject is the study of so-called Transmission eigenvalues that naturally arise in the analysis of inverse medium problems and particularly in the theory behind sampling methods. These frequencies can be seen as the extension of the notion of resonant frequencies for impenetrable objects to the case of penetrable media. Our developments of sampling methods lead us to discover that transmission eigenvalues can be reconstructed from multi-frequency and multistatic measurements. The spectrum formed by these special frequencies can be related to the materiel properties of the medium and therefore can be used as a signature that characterizes some aspects of this medium (although a complete answer to this statement is far from being available). More specifically we promote the use of these quantities to obtain qualitative information on changes in the probed domain (as in non destructive testing). The study of transmission eigenvalues has become (as for sampling methods) a well identified branch in the inverse problem community that raises many challenges ranging from purely theoretical questions to numerical schemes and significance for applications.

For the identification problem, one would also like to have information on the physical properties of the targets. Of course optimization methods is a tool of choice for these problems. The application of non linear optimization methods for inverse problems has to be supplemented by regularization strategies. While convergence for Hilbertian regularization is well understood from the theoretical point of view, it is still far from being the case for non Hilbertian norms. For instance regularization strategies that promote sparsity belong to the latter class and is of great interest for inverse problems where the coefficients have singularities (point sources, crack like defects, piecewise constant material properties, etc...). Exploring theoretical and numerical issues raised by these regularization is of interest

for applications. We plan to invest on these issues together with the use of non standard fidelity functional that may help reducing the number of local minima. Combining deterministic techniques with stochastic ones is also an interesting perspective that has not been sufficiently explored in the literature and that we aim at developing. This would allow us to also investigate feasibility of Bayesian inference for these non linear and computationally involving inverse problems. Exploiting neural networks in the design of solutions to inverse problems is major trend in the inverse problem community as in many other scientific area. Using these techniques to directly solve severe ill posed problems, as inverse scattering problems, does not seem a certifiable route. A more reasonable approach to benefit from the ability of these networks to encode high dimensional complex non linear functional would be to use them for automatically adjusting deterministic optimization parameters such as descent steps and/or regularization parameters (or priors). This also holds true for their use in the sampling methods invoked above and we shall start by exploring this combination first.

From the practical point of view, the major limitation of sampling methods would be the need of a large amount of data to achieve a reasonable accuracy. On the other hand, optimization methods do not suffer from this constrain but they require good initial guess to ensure convergence and reduce the number of iterations. Therefore it seems natural to try to combine the two class of methods in order to calibrate the balance between cost and precision.

Independently from the formulation of the optimization problem, the efficiency of inversion algorithm associated with this formulation greatly depends on the efficiency of the forward solver. Our team has already made significant contributions in acceleration techniques for solutions to the forward problem (waves and diffusion). We developed strong expertise in H-matrix compression and combination with other acceleration techniques such as FFT and fast multipole methods for wave problems. Fructifying this into the solution to large scale inverse problems in link with geophysical application or non destructive testing is promising for obtaining feasible inversion algorithm for the full non linear inverse problem. Domain decomposition technique is yet another expertise that we have developed and would like to explore for accelerating the solution of the forward and inverse problems. For the latter, methodologies where iterations on the inversion parameter and the forward and adjoint problems are combined merit investigations. Several approaches have been proposed in the literature, but the convergence of these schemes and their efficiency are not yet clear in the context of inverse ill-posed problems.

Although a major focus will be given to applications and methodologies that can be of interest for EDF, the contours of the IDEFIX research team include topics that may go beyond that perspective.

In particular we are also interested in applications involving the imaging of biological tissues with the technique of Diffusion Magnetic Resonance Imaging (DMRI). Roughly speaking, DMRI gives a measure of the average distance travelled by water molecules in the imaged medium and can give useful information on cellular structure and structural change when the medium is biological tissue. In particular, we would like to infer from DMRI measurements changes in tissue and cellular structure occurring under various physiological or pathological conditions, as well generally the cell morphology in the region of interest. The main challenges here are: 1) to model correctly the measured signals using diffusive-type evolution equations, 2) to handle numerically the geometrical complexity of biological issue; 3) to use the first two to identify physically relevant parameters from the measurements. There are two main groups of approaches to the first two challenges. The first one relies on using random walkers to mimic the diffusion process in a given geometrical configuration. The second one exploits the model given by the Bloch-Torrey partial differential equation, which describes the evolution of the complex transverse water proton magnetization under the influence of diffusion-encoding magnetic field gradients pulses. We primarily work in the simulation and analysis of the Bloch-Torrey partial differential equation in complex geometries, in other words, we follow the second group of approaches. For the third challenge, we are particularly interested in constructing reduced models of the multiple-compartment Bloch-Torrey model using homogenization methods. To solve difficult problems in diffusion MRI on realistic cellular geometries, we use a variety of approaches including finite elements discretization, Laplace eigenfunctions, and machine learning, coupled with advanced techniques such as HPC, low rank approximations, and layer potential representations.



## 4 Application domains

### 4.1 Eddy Current Imaging for steam generator and rotating machine

Eddy Current is an approximation of Maxwell system at low frequency. Probe that works in that quasi-stationary regime are commonly used in non destructive testing. We are interested in inverse problems for these type of measurement in steam generator and more recently for rotating machine.

Steam generators are critical components in nuclear power plants. For a sake of radioactive safety, the water flow (called the primary fluid) which ensures the cooling of the core reactor is separated from the water flow (called the secondary fluid) which is transformed in steam to generate electricity: the heat must be transferred via the steam generator. The primary fluid circulates in tubes with U-shape while the secondary fluid rises up in the steam generator along these tubes.

Without disassembling the steam generator, the lower part of the U-tubes is inaccessible for normal inspections. Therefore, a non-destructive examination procedure, called eddy current testing (ECT), is usually used to detect the presence of deposits. In an ECT, one introduces a probe consisting of coils of wire in the tube that deliver electromagnetic excitation at low frequencies (eddy current regime) and measure induced currents by the external media (ECT signals). The design of robust and reliable PDE based inversion methods to analyze ECT signals is a long term ongoing project with and within EDF R&D. We first developed and analyzed the simplified setting of axisymmetric geometries which allows to model the problem in 2D and to test various options for the direct and inverse algorithms and in particular an algorithm based on the Level Set method. We start extending this work to 3D inversions for various types of probes (with angular resolutions) and various types of defects. And will pursue this in the future in order to treat real data with multiple defects (i.e. cracks, deposit, thin deposit).

Finally related to Eddy Current modeling, we work on low frequency/quasi-stationary Maxwell system in alternators and engines. On those systems EDF is trying to assess from measurements the exact geometry and the physical properties which have potentially (parametric) non linear constitutive laws. A collaboration on this topic with the EDF team that contributes to code CARMEL. Together with the same team and others, we submit a project on an experimental facilities for eddy current where we will contribute on the calibration of the data using inverse problems methodology and extension to geometry not limited to pipes.

### 4.2 Non destructive testing of concrete-like material

Concrete is a widely used material thanks to its appealing (when reinforced) properties and its manageable cost. However, it is a very complex material: highly heterogeneous, multiscale, evolving over time, etc. Yet, it has to be inspected to ensure that the structure is safe and especially when this structure is part of sensitive infrastructures such as power plants. Using mechanical waves to inspect concrete is widely used in practice but many aspects still constitute very challenging problems due to the complex properties of the material. Several defects are of interest and measurements might vary with respect to the target. Imaging are very difficult with conventional techniques due to potentially low contrast and complicated structure (i.e. reinforcing bar, metallic liners). Sampling methods are able to tackle this type of problem by integrating the information on the a priori structure of the geometry. Other methods such as homogenization could be a valuable approached for identifying this type of defects.

The region between air and the first reinforcement steel bar are of primary importance because it shields the steel from the exterior and therefore prevents corrosion. To inspect this area, engineers rely on surface wave techniques similar to Multi-channel Analysis of Surface Waves (MASW) in geophysics. However, concrete is not a layered material and therefore interpretation of this type of measurements is not easy. Basically this method constructs the dispersion curve of surface waves through data processing and then uses this information to invert a layered model for the material. This procedure shares similarities with spectral signature identification and it would be interesting to further clarify this link in order to obtain better interpretation (or reformulation) of MASW in this context.

Similarly to ultrasonic waves, electromagnetic waves are generally used to inspect concrete type materials, we will be involved in imaging complex structure with radar type measurements. A more challenging application is to determine the electromagnetic properties of the material and relate them to the hydration of the cement paste. MASW analysis of the measurements are also consider in this setting

similarly to mechanical waves.

### 4.3 Subsurface imaging

Subsurface imaging up to one hundred meters is of primary importance for power plant safety assessment with respect to seismic activity. Issues range from testing the interface between the bedrock and the bottom of dam, to quantitative map and detection of fracture underneath nuclear power plant and imaging of the sea bed to evaluate the feasibility of construction of offshore wind turbine. Earth imaging is a well studied area but primarily at medium to long distance thanks to oil industry and seismology. However subsurface imaging is not very common and has its own difficulties due to the heterogeneous structure of the soil and the higher frequency of the wave needed to have a satisfactory resolution.

Dykes are structures that are difficult to inspect but are of primary importance for the safety of power plant and MASW is usually applied for that purpose. Using data processing techniques dispersion curves are extracted from recorded surface waves. Engineers then use semi-analytic model of dispersion curves for stratified media to obtain an subsurface image. Extending this type of methods to non-stratified media is challenging and it would be interesting to see how it compares with classical optimization based inversion methods.

As for concrete imaging mechanical, electromagnetic waves and conductivity are the various physics used to probe the medium. Both shared heterogeneous physical properties, embedded in potentially complex geometries and seek unknown of several types and are applications less investigated than medical, seismic or metallic imaging. As research on measurements techniques are carried by the same group in EDF R&D PRISME we will seek for synergy between these two fields.

### 4.4 Applications of concepts related to invisibility for finite set of measurements

As mentioned above, in the team we have developed different approaches to construct obstacles which are invisible for imaging techniques with a finite number of measurements. In what we did, obstacles are invisible at a fixed frequency. It would be important to study more the sensitivity of the invisibility results with respect to the frequency. In the construction algorithms, can we add constraints so that invisibility remains robust to the frequency? In our studies, we have mainly focused our attention on acoustics in waveguides. It would be interesting to investigate electromagnetic and elasticity problems. Moreover, it would be very instructive to investigate how the invisible objects we design theoretically and numerically behave in practice. To proceed, we contemplate to work with experimentalists at the Acoustic Department of the University of Le Mans with whom we are in contact.

Until now, we have only constructed obstacles which are invisible in time-harmonic regime. It would be interesting to study what can be done for time dependent problems. Maybe the first question is as follows: imagine that a time dependent source term is given (a pulse), how to design the geometry so that the signal passes through the structure as if they were no defect? For the applications, one can think to the optimal design of a stent to fix a damaged coronary artery. It is known that an inappropriate shape and material for the stent can produce scattered waves which are harmful for the cardiac muscle.

In the physical community, spectacular advances have recently been made in the development of a new field called "wave front shaping". For a given unknown complex scatterer, the goal is to find the best entrance signal to obtain a given physical property (focalisation of the energy, good transmission, ...). We emphasize that in this context, optimization techniques cannot be used because the scatterer is complex and a priori unknown. This point of view is different from the one considered up to now in the team. Indeed, in our case, the entrance signal is given and we look for the scatterer to have invisibility. It would be interesting to study the connections existing between the two approaches.

Another area which would benefit from the expertise of the team is the design of barriers that would isolate critical buildings of nuclear power plant from seismic solicitation. This topic is directly related to the partial invisibility exposed above. Indeed, for this application, complete cloaking is not necessary as it is enough to transfer the seismic solicitation to other area in space and to reduce the maximal solicitation on specific modes of the structure (either by spreading the solicitation in time or by transferring the solicitation to a more robust mode of vibration). This boils down to design structures ensuring zero transmission of energy between different given modes, a question we also consider in our works dealing with waves propagation in waveguides. This problematic has not yet been planned by EDF in the short

term perspectives but we believe it is worth exploring as there already exist realistic experiments of such cloaking constructions formed by stilts embedded in the soil.

#### 4.5 Other potential applications of interest for EDF

One way to measure flow rate in stationary environment is to use ultrasound measurements. Ultrasound propagates faster in the direction of the flow and slower in the opposite direction. This principle has been used to measure flow rate in pipes with relatively clear water. It is also used for water carrying heterogeneities, like bubbles, sand, stone... but with an experimental approach, with few understanding of the limits of the method. Being able to model the propagation in such an heterogeneous medium and to adapt the analysis of the data would not only improve current measurement techniques but also give tools to know a priori the method limits. There are issue in hydraulic power plants to assess the presence of obstacles or objects in moving fluid using SONAR type measurements. Depending on the application it is not clear yet if the fluid motion could be neglect and it will be the purpose of further research. These applications would be a promising first step to open our expertise towards fluid related problems, which might have large perspectives in our collaboration with EDF. Similarly to non destructive testing of concrete, other approaches rely on electromagnetic or passive measurements (sources of sound locations) in order to assess the flow rate.

#### 4.6 Diffusion MRI

The diffusion magnetic resonance imaging signal arising from biological tissues can be numerically simulated by solving the Bloch–Torrey partial differential equation. Numerical simulations can facilitate the investigation of the relationship between the diffusion MRI signals and cellular structures. With the rapid advance of available computing power, the diffusion MRI community has begun to employ numerical simulations for model formulation and validation, as well as for imaging sequence optimization. For example, in collaboration with CHU de Rennes and the EMPENN team at Inria Rennes, we aim to develop a novel diffusion MRI sequence, optimized towards clinical feasibility, that can contribute to defining practically obtainable and robust imaging biomarkers of chronic inflammation in patients with Multiple Sclerosis.

## 5 Highlights of the year

### 5.1 Awards

- L. Chesnel received a grant to spend one month at Isaac Newton Institute for Mathematical Sciences at Cambridge. He took part to the INI Programme "Mathematical theory and applications of multiple wave scattering" in April/May 2023.

## 6 New software, platforms, open data

### 6.1 New software

#### 6.1.1 ECIP

**Name:** Eddy Current Imaging for Pipes

**Keywords:** Inverse problem, Partial differential equation, HPC, Domain decomposition

**Functional Description:** This software identifies deposit on pipes from measurements of eddy current probes. It is based on finite elements and domain decomposition through the softwares HPDDM, PETSc and FreeFEM, for the resolution of the PDE model of the eddy current measurements. It uses an iterative algorithm to identify the deposit properties.

**Contact:** Lorenzo Audibert

**Partner:** Edf

### 6.1.2 SpinDoctor

**Name:** SpinDoctor Diffusion MRI Simulation Toolbox

**Keywords:** MRI, Simulation, Finite element modelling

**Functional Description:** SpinDoctor can be used

1. to solve the Bloch-Torrey PDE to obtain the dMRI signal (the toolbox provides a way of robustly fitting the dMRI signal to obtain the fitted Apparent Diffusion Coefficient (ADC)),
2. to solve the diffusion equation of the H-ADC model to obtain the ADC,
3. a short-time approximation formula for the ADC is also included in the toolbox for comparison with the simulated ADC.

**URL:** <https://github.com/SpinDoctorMRI/>

**Contact:** Jing Rebecca Li

### 6.1.3 CASTOR

**Keyword:** C++

**Functional Description:** The objective of the castor library is to propose high-level semantics, inspired by the Matlab language, allowing fast software prototyping in a low-level compiled language. It is nothing more than a matrix management layer using the tools of the standard C++ library, in different storage formats (full, sparse and hierarchical). Indeed, the use of IDEs 1 such as Xcode, Visual studio, Eclipse, etc. allows today to execute compiled code (C, C++, fortran, etc.) with the same flexibility as interpreted languages (Matlab, Python, Julia, etc.).

A header-only template library for matrix management has been developed based on the standard C++ library, notably the `std::vector` class. Many tools and algorithms are provided to simplify the development of scientific computing programs. Particular attention has been paid to semantics, for a simplicity of use “à la matlab”, but written in C++. This high-level semantic/low-level language coupling makes it possible to gain efficiency in the prototyping phase, while ensuring performance for applications. In addition, direct access to data allows users to optimize the most critical parts of their code in native C++. Finally, complete documentation is available, as well as continuous integration unit tests. All of this makes it possible to meet the needs of teaching, academic issues and industrial applications at the same time.

The castor library provides tools to :

create and manipulate dense, sparse and hierarchical matrices  
make linear algebra computations based on optimized BLAS library  
make graphical representations based on VTK library  
These tools are used by applicative projects :

finite and boundary element method using Galerkin approximation  
analytical solutions for scattering problems

**URL:** <https://leprojetcastor.gitlab.labos.polytechnique.fr/castor/#>

**Contact:** Matthieu Aussal

## 7 New results

### 7.1 Fast data driven imaging methods

**Participants:** Lorenzo Audibert, Lucas Chesnel, Housseem Haddar, Nouha Jenhani, Hadrien Montanelli, Fabien Pourre, Jean-Marie Henault.

### 7.1.1 Ultrasonic imaging in highly heterogeneous backgrounds

H. Haddar, F. Pourahmadian

This work formally investigates the differential evolution indicators as a tool for ultrasonic tracking of elastic transformation and fracturing in randomly heterogeneous solids. Within the framework of periodic sensing, it is assumed that the background at time  $t_0$  contains (i) a multiply connected set of viscoelastic, anisotropic, and piece-wise homogeneous inclusions, and (ii) a union of possibly disjoint fractures and pores. The support, material properties, and interfacial condition of scatterers in (i) and (ii) are unknown, while elastic constants of the matrix are provided. The domain undergoes progressive variations of arbitrary chemo-mechanical origins such that its geometric configuration and elastic properties at future times are distinct. At every sensing step  $t_0, t_1, \dots$ , multi-modal incidents are generated by a set of boundary excitations, and the resulting scattered fields are captured over the observation surface. The test data are then used to construct a sequence of wavefront densities by solving the spectral scattering equation. The incident fields affiliated with distinct pairs of obtained wavefronts are analyzed over the stationary and evolving scatterers for a suit of geometric and elastic evolution scenarios entailing both interfacial and volumetric transformations. The main theorem establishes the invariance of pertinent incident fields at the loci of static fractures and inclusions between a given pair of time steps, while certifying variation of the same fields over the modified regions. These results furnish a basis for theoretical justification of differential evolution indicators for imaging in complex composites which, in turn, enable the exclusive tomography of evolution in a background endowed with many unknown features [19].

### 7.1.2 Sampling method in linear elasticity for concrete: gravel honeycomb

L. Audibert, H. Haddar, JM Henault, F. Pourre

We consider the propagation of elastic waves at a given wavenumber in a penetrable medium with different levels of complexity ranging from an unbounded homogeneous medium to a partially bounded medium with or without a microstructure mimicking aggregates in concrete. In those various configurations we studied the Linear Sampling Method for imaging gravel honeycomb defects. The numerical solver is based on FreeFEM together with PETSc to perform parallel computations. Different source modeling has been investigated for point sources. The numerical results show satisfactory reconstruction in some idealistic setting. This is preparatory work before handling experimental data on mockup.

### 7.1.3 Differential imaging in periodic media

Y. Boukari, H. Haddar, N. Jenhani

We consider a problem of nondestructive testing of an infinite periodic penetrable layer using acoustic waves. This is an important problem with growing interest since periodic structures are part of many fascinating modern technological designs with applications in (bio)engineering and material sciences. In many sophisticated devices the periodic structure is complicated or difficult to model mathematically, hence evaluating its Green's function which is the fundamental tool of many imaging methods, is computationally expensive or even impossible. On the other hand, when looking for flows in such complex media, the option of reconstructing everything, i.e. both periodic structure and the defects, may not be viable. We proposed in an earlier work an approach which provides a criteria to reconstruct the support of local anomalies without knowing explicitly or reconstructing the periodic healthy background. We provide a theoretical justification of this method that avoids assuming that the local perturbation is also periodic. Our theoretical framework uses functional spaces with continuous dependence with respect to the Floquet-Bloch variable. The corner stone of the analysis is the justification of the Generalized Linear Sampling Method in this setting for a single Floquet-Bloch mode [10].

### 7.1.4 Time-vs. frequency-domain inverse elastic scattering: Theory and experiment

H. Haddar, X. Liu, F. Pourahmadian, J. Song

This study formally adapts the time-domain linear sampling method (TLSM) for ultrasonic imaging of stationary and evolving fractures in safety-critical components. The TLSM indicator is then applied to the laboratory test data and the obtained reconstructions are compared to their frequency-domain counterparts. The results highlight the unique capability of the time-domain imaging functional for

high-fidelity tracking of evolving damage, and its relative robustness to sparse and reduced-aperture data at moderate noise levels. A comparative analysis of the TLSM images against the multifrequency LSM maps further reveals that thanks to the full-waveform inversion in time and space, the TLSM generates images of remarkably higher quality with the same dataset [18].

### 7.1.5 The linear sampling method for random sources

J. Garnier, H. Haddar, H. Montanelli

We present an extension of the linear sampling method for solving the sound-soft inverse acoustic scattering problem with randomly distributed point sources. The theoretical justification of our sampling method is based on the Helmholtz–Kirchhoff identity, the cross-correlation between measurements, and the volume and imaginary near-field operators, which we introduce and analyze. Implementations in MATLAB using boundary elements, the SVD, Tikhonov regularization, and Morozov’s discrepancy principle are also discussed. We demonstrate the robustness and accuracy of our algorithms with several numerical experiments in two dimensions [16].

## 7.2 Transmission eigenvalues, Stability

**Participants:** Lorenzo Audibert, Lucas Chesnel, Housseem Haddar, Amal Labidi, Fabien Pourre.

### 7.2.1 Asymptotic Expansion of Transmission Eigenvalues for Anisotropic Thin Layers

H. Boujlida, H. Haddar, M. Khenissi

We study the asymptotic expansion of transmission eigenvalues for anisotropic thin layers. We establish a rigorous second order expansion for simple transmission eigenvalues with respect to the thickness of the layer. The convergence analysis is based on a generalization of Osborn’s Theorem to non-linear eigenvalue problems by Moskow [19]. We also provide formal derivation in more general cases validating the obtained theoretical result. [9].

### 7.2.2 Stability estimate for an inverse problem for the time harmonic magnetic schrödinger operator from the near and far field pattern

M. Bellassoued, H. Haddar, A. Labidi

We derive conditional stability estimates for inverse scattering problems related to time harmonic magnetic Schrödinger equation. We prove logarithmic type estimates for retrieving the magnetic (up to a gradient) and electric potentials from near field or far field maps. Our approach combines techniques from similar results obtained in the literature for inhomogeneous inverse scattering problems based on the use of geometrical optics solutions. [8].

### 7.2.3 Generalized impedance boundary conditions with vanishing or sign-changing impedance

L. Bourgeois, L. Chesnel

We consider a Laplace type problem with a generalized impedance boundary condition of the form  $\partial_\nu u = -\partial_x(g\partial_x u)$  on a flat part  $\Gamma$  of the boundary. Here  $\nu$  is the outward unit normal vector to  $\partial\Omega$ ,  $g$  is the impedance parameter and  $x$  is the coordinate along  $\Gamma$ . Such problems appear for example in the modelling of small perturbations of the boundary. In the literature, the cases  $g = 1$  or  $g = -1$  have been investigated. In this work, we address situations where  $\Gamma$  contains the origin and  $g(x) = 1_{x>0}(x)x^\alpha$  or  $g(x) = -\text{sign}(x)|x|^\alpha$  with  $\alpha \geq 0$ . In other words, we study cases where  $g$  vanishes at the origin and changes its sign. The main message is that the well-posedness in the Fredholm sense of the corresponding problems depends on the value of  $\alpha$ . For  $\alpha \in [0, 1)$ , we show that the associated operators are Fredholm of index zero while it is not the case when  $\alpha = 1$ . The proof of the first results is based on the reformulation as 1D problems combined with the derivation of compact embedding results for the functional spaces involved in the analysis. The proof of the second results relies on the computation of singularities and

the construction of Weyl's sequences. We also discuss the equivalence between the strong and weak formulations, which is not straightforward. Finally, we provide simple numerical experiments which seem to corroborate the theorems. [32]

#### 7.2.4 Reconstruction of averaging indicators for highly heterogeneous media

L. Audibert, H. Haddar, F. Poure

We propose a new imaging algorithm capable of producing quantitative indicator functions for unknown and possibly highly oscillating media from multistatic far field measurements of scattered fields at a fixed frequency. The algorithm exploits the notion of modified transmission eigenvalues and their determination from measurements. We propose in particular the use of a new modified background obtained as the limit of a metamaterial background. It has the specificity of having a unique non trivial eigenvalue, which is particularly suited for the proposed imaging procedure. We show the efficiency of this new algorithm on some 2D experiments and emphasize its superiority with respect to some classical approaches such as the Linear Sampling Method.

### 7.3 Propagation of waves in waveguides

**Participants:** Lucas Chesnel.

#### 7.3.1 Acoustic waveguide with a dissipative inclusion

L. Chesnel, J. Heleine, S.A. Nazarov, J. Taskinen

We consider the propagation of acoustic waves in a waveguide containing a penetrable dissipative inclusion. We prove that as soon as the dissipation, characterized by some coefficient  $\eta$ , is non zero, the scattering solutions are uniquely defined. Additionally, we give an asymptotic expansion of the corresponding scattering matrix when  $\eta \rightarrow 0^+$  (small dissipation) and when  $\eta \rightarrow +\infty$  (large dissipation). Surprisingly, at the limit  $\eta \rightarrow +\infty$ , we show that no energy is absorbed by the inclusion. This is due to a skin-effect phenomenon and can be explained by the fact that the field no longer penetrates into the highly dissipative inclusion. These results guarantee that in monomode regime, the amplitude of the reflection coefficient has a global minimum with respect to  $\eta$ . The situation where this minimum is zero, that is when the device acts as a perfect absorber, is particularly interesting for certain applications. However it does not happen in general. In this work, we show how to perturb the geometry of the waveguide to create 2D perfect absorbers in monomode regime. Asymptotic expansions are justified by error estimates and theoretical results are supported by numerical illustrations. [12]

#### 7.3.2 Spectrum of the Laplacian with mixed boundary conditions in a chamfered quarter of layer

L. Chesnel, S.A. Nazarov, J. Taskinen

We investigate the spectrum of a Laplace operator with mixed boundary conditions in an unbounded chamfered quarter of layer. This problem arises in the study of the spectrum of the Dirichlet Laplacian in thick polyhedral domains having some symmetries such as the so-called Fichera layer. The geometry we consider depends on two parameters gathered in some vector  $\kappa = (\kappa_1, \kappa_2)$  which characterizes the domain at the edges. We identify the essential spectrum and establish different results concerning the discrete spectrum with respect to  $\kappa$ . In particular, we show that for a given  $\kappa_1 > 0$ , there is some  $h(\kappa_1) > 0$  such that discrete spectrum exists for  $\kappa_2 \in (-\kappa_1, 0) \cup (h(\kappa_1), \kappa_1)$  whereas it is empty for  $\kappa_2 \in [0, h(\kappa_1)]$ . The proofs rely on classical arguments of spectral theory such as the max-min principle. The main originality lies rather in the delicate use of the features of the geometry.[37]

#### 7.3.3 On the breathing of spectral bands in periodic quantum waveguides with inflating resonators

L. Chesnel, S.A. Nazarov

We are interested in the lower part of the spectrum of the Dirichlet Laplacian  $A^\varepsilon$  in a thin waveguide

$\Pi^\varepsilon$  obtained by repeating periodically a pattern, itself constructed by scaling an inner field geometry  $\Omega$  by a small factor  $\varepsilon > 0$ . The Floquet-Bloch theory ensures that the spectrum of  $A^\varepsilon$  has a band-gap structure. Due to the Dirichlet boundary conditions, these bands all move to  $+\infty$  as  $O(\varepsilon^{-2})$  when  $\varepsilon \rightarrow 0^+$ . Concerning their widths, applying techniques of dimension reduction, we show that the results depend on the dimension of the so-called space of almost standing waves in  $\Omega$  that we denote by  $X_\dagger$ . Generically, i.e. for most  $\Omega$ , there holds  $X_\dagger = \{0\}$  and the lower part of the spectrum of  $A^\varepsilon$  is very sparse, made of bands of length at most  $O(\varepsilon)$  as  $\varepsilon \rightarrow 0^+$ . For certain  $\Omega$  however, we have  $\dim X_\dagger = 1$  and then there are bands of length  $O(1)$  which allow for wave propagation in  $\Pi^\varepsilon$ . The main originality of this work lies in the study of the behaviour of the spectral bands when perturbing  $\Omega$  around a particular  $\Omega_\star$  where  $\dim X_\dagger = 1$ . We show a breathing phenomenon for the spectrum of  $A^\varepsilon$ : when inflating  $\Omega$  around  $\Omega_\star$ , the spectral bands rapidly expand before shrinking. In the process, a band dives below the normalized threshold  $\pi^2/\varepsilon^2$ , stops breathing and becomes extremely short as  $\Omega$  continues to inflate. [35]

### 7.3.4 Spectrum of the Dirichlet Laplacian in a thin cubic lattice

L. Chesnel, S.A. Nazarov

We give a description of the lower part of the spectrum of the Dirichlet Laplacian in an unbounded 3D periodic lattice made of thin bars (of width  $\varepsilon \ll 1$ ) which have a square cross section. This spectrum coincides with the union of segments which all go to  $+\infty$  as  $\varepsilon$  tends to zero due to the Dirichlet boundary condition. We show that the first spectral segment is extremely tight, of length  $O(e^{-\delta/\varepsilon})$ ,  $\delta > 0$ , while the length of the next spectral segments is  $O(\varepsilon)$ . To establish these results, we need to study in detail the properties of the Dirichlet Laplacian  $A^\Omega$  in the geometry  $\Omega$  obtained by zooming at the junction regions of the initial periodic lattice. This problem has its own interest and playing with symmetries together with max-min arguments as well as a well-chosen Friedrichs inequality, we prove that  $A^\Omega$  has a unique eigenvalue in its discrete spectrum, which generates the first spectral segment. Additionally we show that there is no threshold resonance for  $A^\Omega$ , that is no non trivial bounded solution at the threshold frequency for  $A^\Omega$ . This implies that the correct 1D model of the lattice for the next spectral segments is a graph with Dirichlet conditions at the vertices. We also present numerics to complement the analysis. [13]

## 7.4 Analysis of negative metamaterials

**Participants:** Lucas Chesnel.

### 7.4.1 Maxwell's equations with hypersingularities at a negative index material conical tip

A.-S. Bonnet-Ben Dhia, L. Chesnel, M. Rihani

We study a transmission problem for the time harmonic Maxwell's equations between a classical positive material and a so-called negative index material in which both the permittivity  $\varepsilon$  and the permeability  $\mu$  take negative values. Additionally, we assume that the interface between the two domains is smooth everywhere except at a point where it coincides locally with a conical tip. In this context, it is known that for certain critical values of the contrasts in  $\varepsilon$  and in  $\mu$ , the corresponding scalar operators are not of Fredholm type in the usual  $H^1$  spaces. In this work, we show that in these situations, the Maxwell's equations are not well-posed in the classical  $L^2$  framework due to existence of hypersingular fields which are of infinite energy at the tip. By combining the T-coercivity approach and the Kondratiev theory, we explain how to construct new functional frameworks to recover well-posedness of the Maxwell's problem. We also explain how to select the setting which is consistent with the limiting absorption principle. From a technical point of view, the fields as well as their curls decompose as the sum of an explicit singular part, related to the black hole singularities of the scalar operators, and a smooth part belonging to some weighted spaces. The analysis we propose rely in particular on the proof of new key results of scalar and vector potential representations of singular fields. [31]

## 7.5 Diffusion MRI



**Participants:** J.-R. Li, C. Fang, Z. Yang, M. Kchaou.

### 7.5.1 A simulation-driven supervised learning framework to estimate brain microstructure using diffusion MRI

Chengran Fang, Zheyi Yang, Demian Wassermann, Jing-Rebecca Li

We propose a framework to train supervised learning models on synthetic data to estimate brain microstructure parameters using diffusion magnetic resonance imaging (dMRI). Although further validation is necessary, the proposed framework aims to seamlessly incorporate realistic simulations into dMRI microstructure estimation. Synthetic data were generated from over 1,000 neuron meshes converted from digital neuronal reconstructions and linked to their neuroanatomical parameters (such as soma volume and neurite length) using an optimized diffusion MRI simulator that produces intracellular dMRI signals from the solution of the Bloch–Torrey partial differential equation. By combining random subsets of simulated neuron signals with a free diffusion compartment signal, we constructed a synthetic dataset containing dMRI signals and 40 tissue microstructure parameters of 1.45 million artificial brain voxels. To implement supervised learning models we chose multilayer perceptrons (MLPs) and trained them on a subset of the synthetic dataset to estimate some microstructure parameters, namely, the volume fractions of soma, neurites, and the free diffusion compartment, as well as the area fractions of soma and neurites. The trained MLPs perform satisfactorily on the synthetic test sets and give promising in-vivo parameter maps on the MGH Connectome Diffusion Microstructure Dataset (CDMD). Most importantly, the estimated volume fractions showed low dependence on the diffusion time, the diffusion time independence of the estimated parameters being a desired property of quantitative microstructure imaging. The synthetic dataset we generated will be valuable for the validation of models that map between the dMRI signals and microstructure parameters. The surface meshes and microstructures parameters of the aforementioned neurons have been made publicly available.[\[15\]](#)

### 7.5.2 A second order asymptotic model for diffusion MRI in permeable media

Marwa Kchaou, Jing-Rebecca Li

Starting from a reference partial differential equation model of the complex transverse water proton magnetization in a voxel due to diffusion-encoding magnetic field gradient pulses, one can use periodic homogenization theory to establish macroscopic models. A previous work introduced an asymptotic model that accounted for permeable interfaces in the imaging medium. In this paper we formulate a higher order asymptotic model to treat higher values of permeability. We explicitly solved this new asymptotic model to obtain a system of ordinary differential equations that can model the diffusion MRI signal and we present numerical results showing the improved accuracy of the new model in the regime of higher permeability.[\[17\]](#)

### 7.5.3 Incorporating interface permeability into the diffusion MRI signal representation using impermeable Laplace eigenfunctions

Zheyi Yang, Chengran Fang, Jing-Rebecca Li

The complex-valued transverse magnetization due to diffusion-encoding magnetic field gradients acting on a permeable medium can be modeled by the Bloch–Torrey partial differential equation. The diffusion magnetic resonance imaging (MRI) signal has a representation in the basis of the Laplace eigenfunctions of the medium. However, in order to estimate the permeability coefficient from diffusion MRI data, it is desirable that the forward solution can be calculated efficiently for many values of permeability. Approach. In this paper we propose a new formulation of the permeable diffusion MRI signal representation in the basis of the Laplace eigenfunctions of the same medium where the interfaces are made impermeable. Main results. We proved the theoretical equivalence between our new formulation and the original formulation in the case that the full eigendecomposition is used. We validated our method numerically and showed promising numerical results when a partial eigendecomposition is used. Two diffusion MRI sequences were used to illustrate the numerical validity of our new method. Significance. Our approach

means that the same basis (the impermeable set) can be used for all permeability values, which reduces the computational time significantly, enabling the study of the effects of the permeability coefficient on the diffusion MRI signal in the future.[20]

## 7.6 Modelling and HPC for wave propagation problems

**Participants:** Jing Rebecca Li, Marcella Bonazzoli, Housseem Haddar, Hadrien Montanelli.

### 7.6.1 Influence of the partition of unity on SORAS preconditioner

M. Bonazzoli, X. Claeys, F. Nataf, P.-H. Tournier

We have investigated numerically the influence of the choice of the partition of unity on the convergence of the Symmetrized Optimized Restricted Additive Schwarz (SORAS) preconditioner for the heterogeneous reaction-convection-diffusion equation. Previously, we had analyzed the convergence of this overlapping domain decomposition preconditioner for generic non self-adjoint or indefinite problems, like the reaction-convection-diffusion equation. In the numerical experiments, we had noticed that the number of iterations for convergence of preconditioned GMRES appeared not to vary significantly when increasing the overlap width. In this work we show that actually this is due to the particular choice of the partition of unity for the preconditioner, and we study the dependence on the overlap and on the number of subdomains for two kinds of partition of unity. Our numerical investigation shows that the second kind of partition of unity, which is non-zero in the interior of the whole overlapping region, generally improves the iteration counts obtained with the first kind of partition of unity, whose gradient is zero on the subdomain interfaces. Moreover, the first kind of partition of unity, which would be the natural choice for ORAS solver instead, yields for SORAS preconditioner iterations counts that do not vary significantly when increasing the overlap width. A proceedings on this topic [22] have been published.

### 7.6.2 Multi-domain FEM-BEM coupling for acoustic scattering

M. Bonazzoli, X. Claeys

We model time-harmonic acoustic scattering by an object composed of piece-wise homogeneous parts and an arbitrarily heterogeneous part. We propose and analyze new formulations that couple, adopting a Costabel-type approach, boundary integral equations for the homogeneous subdomains with domain variational formulations for the heterogeneous subdomain. This is an extension of Costabel FEM-BEM coupling to a multi-domain configuration, with cross-points allowed, i.e. points where three or more subdomains abut. While generally just the exterior unbounded subdomain is treated with the BEM, here we wish to exploit the advantages of BEM whenever it is applicable, that is, for all the homogeneous parts of the scattering object. Our formulation is based on the multi-trace formalism, which initially was introduced for acoustic scattering by piece-wise homogeneous objects; here we allow the wavenumber to vary arbitrarily in a part of the domain. We prove that the bilinear form associated with the proposed formulation satisfies a Gårding coercivity inequality, which ensures stability of the variational problem if it is uniquely solvable. We identify conditions for injectivity and construct modified versions immune to spurious resonances. An article on this topic is under review [29].

### 7.6.3 Computing singular and near-singular integrals over curved boundary elements: The strongly singular case

H. Montanelli, F. Collino, H. Haddar

We present algorithms for computing strongly singular and near-singular surface integrals over curved triangular patches, based on singularity subtraction, the continuation approach, and transplanted Gauss quadrature. We demonstrate the accuracy and robustness of our method for quadratic basis functions and quadratic triangles by integrating it into a boundary element code and solving several scattering problems in 3D. We also give numerical evidence that the utilization of curved boundary elements enhances computational efficiency compared to conventional planar elements [38].

## 7.7 Optimization based inversion methods

**Participants:** Lorenzo Audibert, Marcella Bonazzoli, Mohamed Aziz Boukraa, Housseem Haddar, Tuan Anh Vu, Xiaoli Liu, Denis Vautrin.

### 7.7.1 On the convergence analysis of one-shot inversion methods

M. Bonazzoli, H. Haddar, T. A. Vu

When an inverse problem is solved by a gradient-based optimization algorithm, the corresponding forward and adjoint problems, which are introduced to compute the gradient, can be also solved iteratively. The idea of iterating at the same time on the inverse problem unknown and on the forward and adjoint problem solutions yields to the concept of one-shot inversion methods. We are especially interested in the case where the inner iterations for the direct and adjoint problems are incomplete, that is, stopped before achieving a high accuracy on their solutions. Here, we focus on general linear inverse problems and generic fixed-point iterations for the associated forward problem. We analyze variants of the so-called multi-step one-shot methods, in particular semi-implicit schemes with a regularization parameter. We establish sufficient conditions on the descent step for convergence, by studying the eigenvalues of the block matrix of the coupled iterations. Several numerical experiments are provided to illustrate the convergence of these methods in comparison with the classical gradient descent, where the forward and adjoint problems are solved exactly by a direct solver instead. We observe that very few inner iterations are enough to guarantee good convergence of the inversion algorithm, even in the presence of noisy data. An article on this topic is under review [30].

### 7.7.2 Imaging a dam-rock interface with inversion of a full elastic-acoustic model

L. Audibert, M. Bonazzoli, M. A. Boukraa, H. Haddar, D. Vautrin

We are interested in imaging the interface between the concrete structure of hydroelectric dams and the rock foundation using non-destructive seismic waves. We present a geophysical technique for processing seismic measurements to obtain an image of the interface with metric resolution. The proposed technique is based on "Full Waveform Inversion" with a shape optimization approach. Numerical results using synthetic measurements demonstrate the method ability to accurately recover the interface with a limited number of measurement points and in the presence of noise. A proceedings has been submitted to ICIP2024.

### 7.7.3 A combination of Kohn-Vogelius and DDM methods for a geometrical inverse problem

S. Chaabane, H. Haddar, R. Jerbi We consider the inverse geometrical problem of identifying the discontinuity curve of an electrical conductivity from boundary measurements. This standard inverse problem is used as a model to introduce and study a combined inversion algorithm coupling a gradient descent on the Kohn-Vogelius cost functional with a domain decomposition method that includes the unknown curve in the domain partitioning. We prove the local convergence of the method in a simplified case and numerically show its efficiency for some two dimensional experiments [11].

## 8 Bilateral contracts and grants with industry

### 8.1 Bilateral contracts with industry

**Participants:** Housseem Haddar, Lorenzo Audibert.

- Grant associated with one PhD (CIFRE contract) with EDF R&D on imaging with eddy current using inverse problem methods. The student is Morgan Mathevet.

- Grant from France Relance associated with one PostDoc with EDF R&D on imaging the interface between dam and Bedrock. The postdoc is Mohammed Aziz Boukraa
- Grant from France Relance associated with the mise a disposition of a researcher from EDF R&D on all the theme of IDEFIX. The researcher is Lorenzo Audibert.
- Grant from ANRT associated with one CIFRE phd with EDF R&D on imaging cracks and multiple defects with eddy current using inverse problems. The phd candidate is Morgane Mathevet.

## 8.2 Bilateral Grants with Industry

**Participants:** Marcella Bonazzoli, Housseem Haddar, Hadrien Montanelli, Lucas Chesnel.

- Grant from DGA in the framework of the **CIEDS**, with the objective of extending sampling methods to passive imaging and imaging in a cluttered media (2021-2024). It partly served financing the Postdoc work of H. Montanelli (2022).
- M. Bonazzoli and L. Chesnel are members of ElectroMATH project (Electromagnetic wave propagation in complex media and configurations, 2022-2026), granted by **CIEDS** (IP Paris-AID), coordinated by P. Ciarlet and A. Modave.

## 9 Dissemination

### 9.1 Promoting scientific activities

#### 9.1.1 Scientific events: organisation

##### Member of the organizing committees

- L. Chesnel co-organizes the seminar common to the three teams IDEFIX-MEDISIM-POEMS.
- L. Chesnel participates to the organization of the "Journées Ondes des Poètes 2024" to celebrate the 60th birthday of A.-S. Bonnet-BenDhia, E. Bécache, C. Hazard and E. Lunéville.
- M. Bonazzoli (until July 2023) and H. Montanelli (since September 2023) organize the group seminar of IDEFIX team.
- M. Bonazzoli (with F. Bonizzoni, I. Mazzieri and V. Nikolić) organized a minisymposium at **ENU-MATH 2023** (Lisbon, September 2023).
- H. Haddar co-organized with F. Cakoni four minisymposia sessions at AIP 2023, Germany.

#### 9.1.2 Scientific events: selection

- L. Chesnel and H. Haddar are members of the scientific committee of the Waves conference.
- L. Audibert (2023 - ) is president of the scientific committee of the school CEA-EDF-INRIA.

##### Member of the editorial boards

- H. Haddar is member of the editorial board of the journals : Inverse Problems and Siam Journal of Mathematical Analysis

**Reviewer - reviewing activities** We reviewed papers for international journals in the main scientific themes of the team.

### 9.1.3 Invited talks

- H. Montanelli, New Jersey Institute of Technology, Fluid Mechanics and Waves Seminar, November 2023.
- H. Montanelli, Congress of Young Researchers in Mathematics and Applications, Gif-sur-Yvette, September 2023.
- H. Montanelli, Numerical Analysis in the 21st Century, Oxford, August 2023.
- H. Montanelli, Inria & Côte d'Azur University, FACTAS Seminar, June 2023.
- H. Montanelli, SIAM CSE 2023, Amsterdam, February 2023.
- H. Montanelli, Orléans University, Applied Analysis Seminar, February 2023.
- H. Montanelli, University of Pau, Mathematics and Applications Seminar, February 2023.
- H. Montanelli, Aix-Marseille University, Applied Analysis Seminar, January 2023.
- L. Chesnel, ANEDP seminar, Lille, December 2023.
- L. Chesnel, thematic day PLASMON 2023, Marseille, December 2023.
- L. Chesnel, LMA seminar, Marseille, October 2023.
- L. Chesnel, LMS-Bath Symposium "Operators, Asymptotics, Waves", Bath, July 2023.
- L. Chesnel, INI Programme Mathematical theory and applications of multiple wave scattering, Cambridge, May 2023.
- L. Chesnel, Maxwell institute applied and computational mathematics seminar, Edinburgh, May 2023.
- M. Bonazzoli, CSE23, SIAM Conference on Computational Science and Engineering 2023, Amsterdam, The Netherlands, February 2023.
- H. Haddar, Invited plenary talk at AIP 2023, Gottingen, Germany.
- H. Haddar, LAMHA seminar, June 2023, Sfax, Tunisia.
- H. Haddar, Invited talk at a minisymposium in IEEE CAMA, 2023, Genova, Italy.
- L. Audibert, AIP 2023, Gottingen, Germany, September 2023.

### 9.1.4 Research administration

- M. Bonazzoli is the International partnerships Scientific Correspondent for Inria Saclay.
- M. Bonazzoli took part in Jul. 2023 to the prize committee for **Prix Junior Maryam Mirzakhani** awarded by Fondation Mathématique Jacques-Hadamard (FMJH) to two young female students for a mathematics project.
- M. Bonazzoli is a volunteer member of **Opération Postes** (newsletter and website, which gathers detailed information about the French competitive selections for permanent positions in Mathematics and Informatics, supported by the French academic societies SMAI, SAGIP, SFdS, SIF, and SMF).
- J.-R. Li is a member of INRIA Commission d'Evaluation, 2015-2023.
- H. Haddar is a member of the BCEP of INRIA Saclay.

## 9.2 Teaching - Supervision - Juries

### 9.2.1 Teaching

- Doctorat: H. Haddar et L. Audibert, Inverse problems: Algorithms and Applications. Executive Education, Ecole Polytechnique.
- Master: H. Montanelli, Modal - Modélisation mathématique par la démarche expérimentale, 2nd year of École Polytechnique, creation and supervision of a project for five students.
- Bachelor: H. Montanelli, Commande des systèmes dynamiques, 1st year of ENSTA Paris, 12 TD hours.
- Master: L. Chesnel, Analyse variationnelle des équations aux dérivées partielles, 2nd year of Ecole Polytechnique, 20 TD hours.
- Master: L. Chesnel, Modal - Modélisation mathématique par la démarche expérimentale, 2nd year of École Polytechnique, creation and supervision of a project for two students.
- Bachelor: L. Chesnel, Numerical Methods for ODEs, 3rd year of the Bachelor of Ecole Polytechnique, 20 TD hours.
- Master: M. Bonazzoli, Mathematics for data science, 1st year of Computer Science Master, Université Paris-Saclay, 21 hours (lessons and TD).
- Master: M. Bonazzoli, La méthode des éléments finis, 2nd year of Engineering School, ENSTA Paris, 12 TD hours.
- Bachelor: M. Bonazzoli, Fonctions de variable complexe, 1st year of Engineering School, ENSTA Paris, 12 TD hours.
- Bachelor: L. Audibert, Introduction à la discrétisation des équations aux dérivées partielles, 1st year of Engineering School, ENSTA Paris, 12 TD hours.
- Licence: H. Haddar, Complex analysis and Elementary tools of analysis for partial differential equations, for students in the first year of Ensta ParisTech curriculum. 37 equivalent TD hours. 2021-present.
- Master: J.-R. Li, Refresher Course in Math and a Project on Numerical Modeling Done in Pairs, The Energy Environment: Science Technology and Management (STEEM) Master Program, Ecole Polytechnique.
- Bachelor: J.-R. Li, Introduction à Matlab, ENSTA Paris.

### 9.2.2 Supervision

- PhD in progress: F. Pourre, Using spectral signature for imaging. (2021-), L. Audibert and H. Haddar.
- PhD in progress: M. Mathevet, Eddy current imaging with inverse problems methods. (2022-), L. Audibert and H. Haddar.
- PhD in progress: A. Parigaux, (2022-), Construction of transparent conditions for electromagnetic waveguides, analysis and applications. (2022-), L. Chesnel and A.S. Bonnet Ben Dhia.
- PhD in progress: T.A. Vu, One-shot inversion methods and domain decomposition (2020-), M. Bonazzoli and H. Haddar.
- PhD in progress: A. Boisneault, Numerical methods and high performance simulation for 3D imaging in complex media, (2023-), M. Bonazzoli (with X. Claeys, Sorbonne Université, and P. Marchand, Inria).

- Master's degree research internship: A. Boisneault, Numerical methods and high performance simulation for 3D imaging in complex media, (Apr.–Aug. 2023), M. Bonazzoli (with X. Claeys, Sorbonne Université, and P. Marchand, Inria).
- Postdoc in progress: M.A. Boukraa, Inverse problem methods for interface imaging: application to a concrete-rock interface for a hydroelectric dam, (2022-), supervisors: M. Bonazzoli, D. Vautrin, collaborators: L. Audibert, H. Haddar, F. Taillade.
- Postdoc in progress: D.Q. Bui, Optimal inverse modeling for GPR imaging, (2023-), L. Audibert, M. Bonazzoli, H. Haddar, P. Marchand, F. Taillade.
- PhD: Zheyi Yang (10/2020 – 12/2023). Numerical methods to estimate brain micro-structure from diffusion MRI data. J.-R. Li.
- PhD: Chengran Fang (10/2019 – 2/2023). Neuron modeling, Bloch-Torrey equation, and their application to brain microstructure estimation using diffusion MRI. J.-R. Li, Co-advisor: Demian Wassermann(MIND, Inria Saclay).
- Master's degree research internship: Mohamed Ahmane, Resolution of the inverse problem of electrical impedance tomography by Bayesian inference. J.-R. Li, Co-advisor: Lisl Weynans (CARMEN, Inria Bordeaux).
- PhD: N. Jenhani, Generalized linear sampling methods for locally perturbed periodic layers, (2020-12/2023) H. Haddar and Y. Boukari.
- PhD: A. Labidi, Inverse scattering problems for the time harmonic magnetic Schrödinger operator. (2020-12/2023) H. Haddar and M. Bellassoued.
- PhD: R. Jerbi, Combined inversion methods for inverse conductivity problems (2020-11/2023) H. Haddar and S. Chaabane.

### 9.2.3 Interventions

- M. Bonazzoli and L. Chesnel were volunteers at Inria stand at Fête de la Science (Institut Polytechnique de Paris), Oct. 2023.
- M. Bonazzoli participated to several speed-meetings with high/middle school students (Rendez-vous des Jeunes Mathématiciennes et Informatiennes at Inria Saclay, Feb. 2023, and at École Normale Supérieure Paris, Dec. 2023; Journée Filles, maths et informatique : une équation lumineuse at École Polytechnique, Mar. 2023) to answer their questions about the studies and career as a mathematician.
- L. Chesnel went to an elementary school class to make an initiation to geometry.
- J.-R. Li participated in Rencontre avec les Bachelor de l'X autour des mathématiques, Nov. 2023, Institut des Hautes Études Scientifiques (IHES).

## 10 Scientific production

### 10.1 Major publications

- [1] L. Audibert, H. Girardon, H. Haddar and P. Jolivet. 'Inversion of Eddy-Current Signals Using a Level-Set Method and Block Krylov Solvers'. In: *SIAM Journal on Scientific Computing* (2023). URL: <https://hal.science/hal-03043491>.
- [2] M. Bonazzoli, H. Haddar and T. A. Vu. *On the convergence analysis of one-shot inversion methods*. 4th July 2023. URL: <https://inria.hal.science/hal-04151014>.
- [3] Y. Boukari, H. Haddar and N. Jenhani. 'Analysis of sampling methods for imaging a periodic layer and its defects'. In: *Inverse Problems* 39.5 (17th Mar. 2023), p. 055001. DOI: [10.1088/1361-6420/ac19a](https://doi.org/10.1088/1361-6420/ac19a). URL: <https://hal.science/hal-03876852>.

- [4] L. Chesnel, J. Heleine, S. A. Nazarov and J. Taskinen. ‘Acoustic waveguide with a dissipative inclusion’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.6 (Nov. 2023), pp. 3585–3613. DOI: [10.1051/m2an/2023070](https://doi.org/10.1051/m2an/2023070). URL: <https://hal.science/hal-04357144>.
- [5] C. Fang, Z. Yang, D. Wassermann and J.-R. Li. ‘A simulation-driven supervised learning framework to estimate brain microstructure using diffusion MRI’. In: *Medical Image Analysis* 90 (Dec. 2023), p. 102979. DOI: [10.1016/j.media.2023.102979](https://doi.org/10.1016/j.media.2023.102979). URL: <https://hal.science/hal-04254343>.
- [6] H. Montanelli, F. Collino and H. Haddar. *Computing singular and near-singular integrals over curved boundary elements: The strongly singular case*. 29th Sept. 2023. URL: <https://hal.science/hal-04224658>.

## 10.2 Publications of the year

### International journals

- [7] L. Audibert, H. Girardon, H. Haddar and P. Jolivet. ‘Inversion of Eddy-Current Signals Using a Level-Set Method and Block Krylov Solvers’. In: *SIAM Journal on Scientific Computing* 45.3 (2023), B366–B389. DOI: [10.1137/20M1382064](https://doi.org/10.1137/20M1382064). URL: <https://hal.science/hal-03043491>.
- [8] M. Bellassoued, H. Haddar and A. Labidi. ‘Stability estimate for an inverse problem for the time harmonic magnetic schrödinger operator from the near and far field pattern’. In: *SIAM Journal on Mathematical Analysis* 55.4 (2023), pp. 2475–2504. DOI: [10.1137/22M1481956](https://doi.org/10.1137/22M1481956). URL: <https://inria.hal.science/hal-03876858>.
- [9] H. Boujlida, H. Haddar and M. Khenissi. ‘Asymptotic Expansion of Transmission Eigenvalues for Anisotropic Thin Layers’. In: *Applicable Analysis* (2023), pp. 1–22. DOI: [10.1080/00036811.2023.2187788](https://doi.org/10.1080/00036811.2023.2187788). URL: <https://hal.science/hal-03876855>.
- [10] Y. Boukari, H. Haddar and N. Jenhani. ‘Analysis of sampling methods for imaging a periodic layer and its defects’. In: *Inverse Problems* 39.5 (17th Mar. 2023), p. 055001. DOI: [10.1088/1361-6420/acc19a](https://doi.org/10.1088/1361-6420/acc19a). URL: <https://hal.science/hal-03876852>.
- [11] S. Chaabane, H. Haddar and R. Jerbi. ‘A combination of Kohn-Vogelius and DDM methods for a geometrical inverse problem’. In: *Inverse Problems* 39.9 (21st July 2023), p. 095001. DOI: [10.1088/1361-6420/ace64a](https://doi.org/10.1088/1361-6420/ace64a). URL: <https://inria.hal.science/hal-03940961>.
- [12] L. Chesnel, J. Heleine, S. A. Nazarov and J. Taskinen. ‘Acoustic waveguide with a dissipative inclusion’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.6 (Nov. 2023), pp. 3585–3613. DOI: [10.1051/m2an/2023070](https://doi.org/10.1051/m2an/2023070). URL: <https://hal.science/hal-04357144>.
- [13] L. Chesnel and S. A. Nazarov. ‘Spectrum of the Dirichlet Laplacian in a thin cubic lattice’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.6 (Nov. 2023), pp. 3251–3273. DOI: [10.1051/m2an/2023082](https://doi.org/10.1051/m2an/2023082). URL: <https://hal.science/hal-04314942>.
- [14] C. Fang, D. Wassermann and J.-R. Li. ‘Fourier representation of the diffusion MRI signal using layer potentials’. In: *SIAM Journal on Applied Mathematics* 83.1 (28th Feb. 2023), pp. 99–121. DOI: [10.1137/21M1439572](https://doi.org/10.1137/21M1439572). URL: <https://hal.science/hal-03940100>.
- [15] C. Fang, Z. Yang, D. Wassermann and J.-R. Li. ‘A simulation-driven supervised learning framework to estimate brain microstructure using diffusion MRI’. In: *Medical Image Analysis* 90 (Dec. 2023), p. 102979. DOI: [10.1016/j.media.2023.102979](https://doi.org/10.1016/j.media.2023.102979). URL: <https://hal.science/hal-04254343>.
- [16] J. Garnier, H. Haddar and H. Montanelli. ‘The linear sampling method for random sources’. In: *SIAM Journal on Imaging Sciences* 16.3 (2023), pp. 1572–1593. DOI: [10.1137/22M1531336](https://doi.org/10.1137/22M1531336). URL: <https://hal.science/hal-03832969>.
- [17] M. Kchaou and J.-R. Li. ‘A second order asymptotic model for diffusion MRI in permeable media’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.4 (July 2023), pp. 1953–1980. DOI: [10.1051/m2an/2023043](https://doi.org/10.1051/m2an/2023043). URL: <https://hal.science/hal-04149697>.
- [18] X. Liu, J. Song, F. Pourahmadian and H. Haddar. ‘Time-vs. frequency-domain inverse elastic scattering: Theory and experiment’. In: *SIAM Journal on Applied Mathematics* 83.3 (2023). DOI: [10.1137/22M1522437](https://doi.org/10.1137/22M1522437). URL: <https://inria.hal.science/hal-03876860>.



- [19] F. Pourahmadian and H. Haddar. ‘Ultrasonic imaging in highly heterogeneous backgrounds’. In: *Proceedings of the Royal Society of London. Series A, Mathematical and physical sciences* 479.2271 (2023). DOI: [10.1098/rspa.2022.0721](https://doi.org/10.1098/rspa.2022.0721). URL: <https://inria.hal.science/hal-03876861>.
- [20] Z. Yang, C. Fang and J.-R. Li. ‘Incorporating interface permeability into the diffusion MRI signal representation while using impermeable Laplace eigenfunctions’. In: *Physics in Medicine and Biology* 68.17 (29th Aug. 2023), p. 175036. DOI: [10.1088/1361-6560/acf022](https://doi.org/10.1088/1361-6560/acf022). URL: <https://hal.science/hal-04254331>.
- [21] Z. Yang, I. Mekkaoui, J. Hesthaven and J.-R. Li. ‘Asymptotic models of the diffusion MRI signal accounting for geometrical deformations’. In: *MathematicS In Action* 12.1 (2023), pp. 65–85. DOI: [10.5802/msia.32](https://doi.org/10.5802/msia.32). URL: <https://hal.science/hal-03939649>.

### Scientific book chapters

- [22] M. Bonazzoli, X. Claeys, F. Nataf and P.-H. Tournier. ‘How does the partition of unity influence SORAS preconditioner?’ In: *Domain Decomposition Methods in Science and Engineering XXVII*. Vol. 149. Lecture Notes in Computational Science and Engineering. Springer Nature Switzerland, 23rd Jan. 2024, pp. 61–68. DOI: [10.1007/978-3-031-50769-4\\_6](https://doi.org/10.1007/978-3-031-50769-4_6). URL: <https://hal.science/hal-03882577>.

### Doctoral dissertations and habilitation theses

- [23] C. Fang. ‘Neuron modeling, Bloch-Torrey equation, and their application to brain microstructure estimation using diffusion MRI’. Université Paris-Saclay, 2nd Feb. 2023. URL: <https://theses.hal.science/tel-04043104>.
- [24] N. Jenhani. ‘Generalized linear sampling methods for locally perturbed periodic layers’. Ecole Nationale d’ingénieur de Tunis, 23rd Dec. 2023. URL: <https://hal.science/tel-04417003>.
- [25] R. Jerbi. ‘Combined inversion methods for inverse conductivity problems’. University of Sfax, 28th Oct. 2023. URL: <https://theses.hal.science/tel-04401679>.
- [26] A. LABIDI. ‘Inverse scattering problems for the time harmonic magnetic Schrödinger operator’. Université Tunis El Manar (Tunisie), 23rd Dec. 2023. URL: <https://theses.hal.science/tel-04404380>.

### Reports & preprints

- [27] L. Audibert, H. Haddar and F. Pourre. *Reconstruction of averaging indicators for highly heterogeneous media*. 2023. URL: <https://inria.hal.science/hal-04223245>.
- [28] L. Baratchart, H. Haddar and C. Villalobos Guillén. *Silent sources on a surface for the Helmholtz equation and decomposition of  $L^2$  vector fields*. 30th Dec. 2023. URL: <https://hal.science/hal-04367726>.
- [29] M. Bonazzoli and X. Claeys. *Multi-domain FEM-BEM coupling for acoustic scattering*. 15th May 2023. URL: <https://hal.science/hal-04098053>.
- [30] M. Bonazzoli, H. Haddar and T. A. Vu. *On the convergence analysis of one-shot inversion methods*. 4th July 2023. URL: <https://inria.hal.science/hal-04151014>.
- [31] A.-S. Bonnet-Ben, L. Chesnel and M. Rihani. *Maxwell’s equations with hypersingularities at a negative index material conical tip*. 3rd May 2023. URL: <https://hal.science/hal-04087360>.
- [32] L. Bourgeois and L. Chesnel. *Generalized impedance boundary conditions with vanishing or sign-changing impedance*. 23rd Sept. 2023. URL: <https://hal.science/hal-04216044>.
- [33] F. Cakoni, H. Haddar and T.-P. Nguyen. *Fast Imaging of Local Perturbations in a Unknown Bi-Periodic Layered Medium*. 2023. URL: <https://inria.hal.science/hal-04223269>.
- [34] L. Chesnel, J. Heleine, S. A. Nazarov and J. Taskinen. *Acoustic waveguide with a dissipative inclusion*. 21st May 2023. URL: <https://hal.science/hal-03711258>.

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- [35] L. Chesnel and S. A. Nazarov. *On the breathing of spectral bands in periodic quantum waveguides with inflating resonators*. 31st Dec. 2023. URL: <https://hal.science/hal-04368072>.
  - [36] L. Chesnel and S. A. Nazarov. *Spectrum of the Dirichlet Laplacian in a thin cubic lattice*. 16th Jan. 2023. URL: <https://hal.science/hal-03940303>.
  - [37] L. Chesnel, S. A. Nazarov and J. Taskinen. *Spectrum of the Laplacian with mixed boundary conditions in a chamfered quarter of layer*. 27th Mar. 2023. URL: <https://hal.science/hal-04048228>.
  - [38] H. Montanelli, F. Collino and H. Haddar. *Computing singular and near-singular integrals over curved boundary elements: The strongly singular case*. 29th Sept. 2023. URL: <https://hal.science/hal-04224658>.