2023
ACTIVITY REPORT

Project-Team

ELAN

modELing the Appearance of Nonlinear phenomena

IN COLLABORATION WITH: Laboratoire Jean Kuntzmann (LJK)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

Contents

Project-Team ELAN

Creation of the Project-Team: 2019 August 01

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1   Team members, visitors, external collaborators

Research Scientists

- Florence Descoubes [Team leader, INRIA, Senior Researcher, HDR]

- Thibaut Metivet [INRIA, Researcher]

- Victor Romero Gramegna [INRIA, ISFP]

PhD Students

- Emile Hohnadel [ENS DE LYON]

- Jean Jouve [ENS RENNES]

- Alexandre Teixeira Da Silva [INRIA]

Technical Staff

- Octave Crespel [ENSIMAG, Engineer, from Sep 2023]

Interns and Apprentices

- Adrien Decosse [ENS Rennes, Intern, from May 2023 until Jul 2023]

Administrative Assistant

- Julia Di Toro [INRIA]

Visiting Scientist

- Rahul Narain [INRIA, from Oct 2023]

2   Overall objectives

ELAN is a young research team of Inria and Laboratoire Jean Kuntzmann (UMR 5224), with an original positioning across Computer Graphics and Computational Mechanics. The team is focussed on the design of predictive, robust, efficient, and controllable numerical models for capturing the shape and motion of visually rich mechanical phenomena, such as the buckling of an elastic ribbon, the flowing of sand, or the entangling of large fiber assemblies. Target applications encompass the digital entertainment industry (e.g., feature film animation, special effects), as well as virtual prototyping for the mechanical engineering industry (e.g., direct and inverse design of textiles and metamaterials, sport performance optimisation, cosmetology); though very different, these two application fields require predictive and scalable models for capturing complex mechanical phenomena at the macroscopic scale. An orthogonal objective is the improvement of our understanding of natural physical and biological processes involving slender structures and frictional contact, through active collaborations with soft matter physicists. To achieve its goals, the team strives to master as finely as possible the entire modeling pipeline, involving a pluridisciplinary combination of scientific skills across Mechanics and Physics, Applied Mathematics, and Computer Science.

3   Research program

Thanks to an original and transverse positioning across Computer Graphics and Computational Mechanics, complemented by tight connections with physicists, our goal is to tackle some challenging numerical modelling issues related to complex macroscopic phenomena characterised by a nonlinear mechanical behaviour and rich geometrical deformations. One major ambition of the ELAN team is to favour interactions between all the relevant communities, with two objectives: 1/ significantly improve our understanding and modelling capabilities of complex mechanical phenomena, in tight connection with physicists, and 2/ better anticipate practical solutions for the wide diversity of exciting applications to come in the near future. We propose in particular to focus on three research axes, detailed below.

3.1   Discrete modelling of slender elastic structures

For the last 15 years, we have investigated new discrete models for solving the Kirchhoff dynamic equations for thin elastic rods [20, 21, 24]. All our models share a curvature-based spatial discretisation, allowing them to capture inextensibility of the rod intrinsically, without the need for adding any kinematic constraint. Moreover, elastic forces boil down to linear terms in the dynamic equations, making them well-suited for implicit integration. Interestingly, our discretisation methodology can be interpreted from two different points-of-views. From the finite-elements point-of-view, our strain-based discrete schemes can be seen as discontinuous Galerkin methods of zero and first orders. From the multibody system dynamics point of view, our discrete models can be interpreted as deformable Lagrangian systems in finite dimension, for which a dedicated community has started to grow recently [47]. We note that adopting the multibody system dynamics point of view helped us formulate a linear-time integration scheme [19], which had only be investigated in the case of multibody rigid bodies dynamics so far.

High-order spatial discretisation schemes for rods, ribbons and shells   Our goal is to investigate similar high-order modelling strategies for surfaces, in particular for the case of inextensible ribbons and shells. Elastic ribbons have been scarcely studied in the past, but they are nowadays drawing more and more the attention from physicists [34, 45]. Their numerical modelling remains an open challenge. In contrast to ribbons, a huge litterature exists for shells, both from a theoretical and numerical viewpoints (see, e.g., [39, 25]). However, no real consensus has been obtained so far about a unified nonlinear shell theory able to support large displacements. In [23] we have started building an inextensible shell patch by taking as degrees of freedom the curvatures of its mid-surface, expressed in the local frame. As in the super-helix model, we show that when taking curvatures uniform over the element, each term of the equations of motion may be computed in closed-form; besides, the geometry of the element corresponds to a cylinder patch at each time step. Compared to the 1D (rod) case however, some difficulties arise in the 2D (plate/shell) case, where compatibility conditions are to be treated carefully. In [2] we have proposed a new, curvature-based discretisation for a developable ribbon (i.e., a narrow plate), which we plan to extend for building an inextensible plate model.

Numerical continuation of rod equilibria in the presence of unilateral constraints   In Alejandro Blumentals' PhD thesis [22], we have adopted an optimal control point of view on the static problem of thin elastic rods, and we have shown that direct discretisation methods [1] are particularly well-suited for dealing with scenarios involving both bilateral and unilateral constraints (such as contact). We would like to investigate how our formulations extend to continuation problems, where the goal is to follow a certain branch of equilibria when the rod is subject to some varying constraints (such as one fixed end being applied a constant rotation). To the best of our knowledge, classical continuation methods used for rods [35] are not able to deal with non-persistent or sliding contact.

3.2   Discrete and continuous modelling of frictional contact

Most popular approaches in Computer Graphics and Mechanical Engineering consist in assuming that the objects in contact are locally compliant, allowing them to slightly penetrate each other. This is the

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[1]Within this optimal control framework, our previous curvature-based methods can actually be interpreted as a special case of direct single shooting methods.

principle of penalty-based methods (or molecular dynamics), which consists in adding mutual repulsive forces of the form $k\,f(\delta)$, where δ is the penetration depth detected at current time step [26, 44]. Though simple to implement and computationally efficient, the penalty-based method often fails to prevent excessive penetration of the contacting objects, which may prove fatal in the case of thin objects as those may just end up traversing each other. One solution might be to set the stiffness factor k to a large enough value, however this causes the introduction of parasitical high frequencies and calls for very small integration steps [18]. Penalty-based approaches are thus generally not satisfying for ensuring robust contact handling.

In the same vein, the friction law between solid objects, or within a yield-stress fluid (used to model foam, sand, or cement, which, unlike water, cannot flow beyond a certain threshold), is commonly modeled using a regularised friction law (sometimes even with simple viscous forces), for the sake of simplicity and numerical tractability (see e.g., [46, 37]). Such a model cannot capture the threshold effect that characterises friction between contacting solids or within a yield-stress fluid. The nonsmooth transition between sticking and sliding is however responsible for significant visual features, such as the complex patterns resting on the outer surface of hair, the stable formation of sand piles, or typical stick-slip instabilities occurring during motion.

The search for a realistic, robust and stable frictional contact method encouraged us to depart from those, and instead to focus on rigid contact models coupled to the exact nonsmooth Coulomb law for friction (and respectively, to the exact nonsmooth Drucker-Prager law in the case of a fluid), which better integrate the effects of frictional contact at the macroscopic scale. This motivation was the sense of the hiring of F. Bertails-Descoubes in 2007 in the Inria/LJK BIPOP team, specialised in nonsmooth mechanics and related convex optimisation methods. In the line of F. Bertails-Descoubes's work performed in the BIPOP team, the ELAN team keeps on including some active research on the finding of robust frictional contact algorithms specialised for slender deformable structures.

Optimised algorithms for large nodal systems in frictional contact   In the fibre assembly case, the resulting mass matrix M is block-diagonal, so that the Delassus operator can be computed in an efficient way by leveraging sparse-block computations [28]. This justifies solving the reduced discrete frictional contact problem where primary unknowns are forces, as usually advocated in nonsmooth mechanics [42]. For cloth however, where primal variables (nodal velocities of the cloth mesh) are all interconnected via elasticity through implicit forces, the method developed above is computationally inefficient. Indeed, the matrix M (only block-sparse, but not block-diagonal) is costly to invert for large systems and its inverse is dense. Recently, we have leveraged the fact that generalised velocities of the system are 3D velocities, which simplifies the discrete contact problem when contacts occur at the nodes. Combined with a multiresolution strategy, we have devised an algorithm able to capture exact Coulomb friction constraints at contact, while retaining computational efficiency [43]. This work also supports cloth self-contact and cloth multilayering. How to enrich the interaction model with, e.g., cohesion, remains an open question. The experimental validation of our frictional contact model is also one of our goals in the medium run.

Continuum modelling of granular and fibrous media   Though we have recently made progress on the continuum formulation and solving of granular materials in Gilles Daviet's PhD thesis [27, 30, 29], we are still far from a continuum description of a macroscopic dry fibrous medium such as hair. One key ingredient that we have not been considering in our previous models is the influence of air inside divided materials. Typically, air plays a considerable role in hair motion. To advance in that direction, we have started to look at a diphasic fluid representation of granular matter, where a Newtonian fluid and the solid phase are fully coupled, while the nonsmooth Drucker-Prager rheology for the solid phase is enforced implicitly [31]. This first approach could be a starting point for modelling immersed granulars in a liquid, or ash clouds, for instance.

A long path then remains to be achieved, if one wants to take into account long fibres instead of isotropic grains in the solid phase. How to couple the fibre elasticity with our current formulation remains a challenging problem.

3.3 Inverse design of slender elastic structures [ERC GEM]

With the considerable advance of automatic image-based capture in Computer Vision and Computer Graphics these latest years, it becomes now affordable to acquire quickly and precisely the full 3D geometry of many mechanical objects featuring intricate shapes. Yet, while more and more geometrical data get collected and shared among the communities, there is currently very little study about how to infer the underlying mechanical properties of the captured objects merely from their geometrical configurations.

An important challenge consists in developing a non-invasive method for inferring the mechanical properties of complex objects from a minimal set of geometrical poses, in order to predict their dynamics. In contrast to classical inverse reconstruction methods, our claim is that 1/ the mere geometrical shape of physical objects reveals a lot about their underlying mechanical properties and 2/ this property can be fully leveraged for a wide range of objects featuring rich geometrical configurations, such as slender structures subject to contact and friction (e.g., folded cloth or twined filaments).

In addition to significant advances in fast image-based measurement of diverse mechanical materials stemming from physics, biology, or manufacturing, this research is expected in the long run to ease considerably the design of physically realistic virtual worlds, as well as to boost the creation of dynamic human doubles.

To achieve this goal, we shall develop an original inverse modelling strategy based upon the following research topics:

Design of well-suited discrete models for slender structures   We believe that the quality of the up-stream, reference physics-based model is essential to the effective connection between geometry and mechanics. Typically, such a model should properly account for the nonlinearities due to large displacements of the structures, as well as to the nonsmooth effects typical of contact and friction.

It should also be parametrised and discretised in such a way that inversion gets simplified mathematically, possibly avoiding the huge cost of large and nonconvex optimisation. In that sense, unlike concurrent methods which impose inverse methods to be compatible with a generic physics-based model, we instead advocate the design of specific physics-based models which are tailored for the inversion process.

More precisely, from our experience on fibre modelling, we believe that reduced Lagrangian models, based on a minimal set of coordinates and physical parameters (as opposed to maximal coordinates models such as mass-springs), are particularly well-suited for inversion and physical interpretation of geometrical data [33, 32]. Furthermore, choosing a high-order coordinate system (e.g., curvatures instead of angles) allows for a precise handling of curved boundaries and contact geometry, as well as the simplification of constitutive laws (which are transformed into a linear equation in the case of rods). We are currently investigating high-order discretisation schemes for elastic ribbons and developable shells [23, 2].

Static inversion of physical objects from geometrical poses   We believe that pure static inversion may by itself reveal many insights regarding a range of parameters such as the undeformed configuration of the object, some material parameters or contact forces.

The typical settings that we consider is composed of, on the one hand, a reference mechanical model of the object of interest, and on the other hand a single or a series of complete geometrical poses corresponding each to a static equilibrium. The core challenge consists in analyzing theoretically and practically the amount of information that can be gained from one or several geometrical poses, and to understand how the fundamental under-determinacy of the inverse problem can be reduced, for each unknown quantity (parameter or force) at play. Both the equilibrium condition and the stability criterion of the equilibrium are leveraged towards this goal. On the theoretical side, we have recently shown that a given 3D curve always matches the centerline of an isotropic suspended Kirchhoff rod at equilibrium under gravity, and that the natural configuration of the rod is unique once material parameters (mass, Young modulus) are fixed [1]. On the practical side, we have recently devised a robust algorithm to find a valid natural configuration for a discrete shell to match a given surface under gravity and frictional contact forces [4]. Unlike rods however, shells can have multiple inverse (natural) configurations. Choosing among the multiple solutions based on some selection criteria is an open challenge. Another open issue,

in all cases, is the theoretical characterisation of material parameters allowing the equilibrium to be stable.

Dynamic inversion of physical objects from geometrical poses   To refine the solution subspaces searched for in the static case and estimate dynamic parameters (e.g., some damping coefficients), a dynamic inversion process accounting for the motion of the object of interest is necessary.

In contrast to the static case where we can afford to rely on exact geometrical poses, our analysis in the dynamic case will have to take into account the imperfect quality of input data with possible missing parts or outliers. One interesting challenge will be to combine our high-order discretised physics-based model together with the acquisition process in order to refine both the parameter estimation and the geometrical acquisition. Our pluridisciplinary work [6] gives encouraging results regarding the ability to recover material parameters and friction coefficient from merely visual observations of elastic bodies in motion.

Experimental validation with respect to real data   The goal will be to confront the theories developed above to real experiments. Compared to the statics, the dynamic case will be particularly involving as it will be highly dependent on the quality of input data as well as the accuracy of the motion predicted by our physics-based simulators. Such experiments will not only serve to refine our direct and inverse models, but will also be leveraged to improve the 3D geometrical acquisition of moving objects. Besides, once validation will be performed, we shall work on the setting up of new non-invasive measurement protocols to acquire physical parameters of slender structures from a minimal amount of geometrical configurations. Our recent publication on validation benchmarks [7] represents a first important milestone towards this research direction.

4   Application domains

4.1   Mechanical Engineering

Many physicists and mathematicians have strived for centuries to understand the principles governing those complex mechanical phenomena, providing a number of continuous models for slender structures, granular matter, and frictional contact. In the XX$^{\text{th}}$ century, industrial applications such as process automatization and new ways of transportation have boosted the fields of Mechanical Engineering and Computer-Aided Design, where material strength, reliability of mechanisms, and safety, standed for the main priorities. Instead, large displacements of structures, buckling, tearing, or entanglement, and even dynamics, were long considered as undesirable behaviors, thus restraining the search for corresponding numerical models.

Only recently, the engineering industry has shown some new and growing interest into the modeling of dynamic phenomena prone to large displacements, contact and friction. For instance, the cosmetology industry is more and more interested in understanding the nonlinear deformation of hair and skin, with the help of simulation. Likewise, auto and aircraft manufacturers are facing new challenges involving buckling or entanglement of thin structures such as carbon or optical fibers; they clearly lack predictive, robust and efficient numerical tools for simulating and optimizing their new manufacturing process, which share many common features with the large-scale simulation scenarii traditionally studied in Computer Graphics applications.

4.2   Computer Graphics

In contrast, Computer Graphics, which has emerged in the 60's with the advent of modern computers, was from the very beginning eager to capture such peculiar phenomena, with the sole aim to produce spectacular images and create astonishing stories. At the origin, Computer Graphics thus drastically departed from other scientific fields. Everyday-life phenomena such as cloth buckling, paper tearing, or hair fluttering in the wind, mostly ignored by other scientists at that time, became actual topics of interest, involving a large set of new research directions to be explored, both in terms of modelling and simulation. Nowadays, although the image production still remains the core activity of the Computer Graphics

community, more and more research studies are directed through the virtual and real prototyping of mechanical systems, notably driven by a myriad of new applications in the virtual try on industry (e.g., hairstyling and garment fitting). Furthermore, the advent of additive fabrication is currently boosting research in the free design of new mechanisms or systems for various applications, from architecture design and fabrication of metamaterials to the creation of new locomotion modes in robotics. Some obvious common interests and approaches are thus emerging between Computer Graphics and Mechanical Engineering, yet the two communities remain desperately compartmentalized.

4.3 Soft Matter Physics

From the physics-based viewpoint, since a few decades a new generation of physicists became interested again in the understanding of such visually fascinating phenomena, and started investigating the tight links between geometry and elasticity [2]. Common objects such as folded or torn paper, twined plants, coiled honey threads, or human hair have thus regained some popularity among the community in Nonlinear Physics [3]. In consequence, phenomena of interest have become remarkably close to those of Computer Graphics, since scientists in both places share the common goal to model complex and integrated mechanical phenomena at the macroscopic scale. Of course, the goals and employed methodologies differ substantially from one community to the other, but showcase some evident complementarity: while computer scientists are eager to learn and understand new physical models, physicists get more and more interested in the numerical tools, in which they perceive not only a means to confirm predictions afterwards, but also a support for testing new hypothesis and exploring scenarios that would be too cumbersome or even impossible to investigate experimentally. Besides, numerical exploration starts becoming a valuable tool for getting insights into the search for analytic solutions, thus fully participating to the modeling stage and physical understanding. However, physicists may be limited to a blind usage of numerical black boxes, which may furthermore not be dedicated to their specific needs. According to us, promoting a science of modeling in numerical physics would thus be a promising and rich avenue for the two research fields. Unfortunately, very scarce cooperation currently exists between the two communities, and large networks of collaboration still need to be set up.

5 Social and environmental responsibility

5.1 Footprint of research activities

The ELAN team is environment-sensitive. Since its creation in 2017, 100% of its research staff moves daily from home to the lab using soft transportation means (biking, public transportation). Intercontinental missions are limited while train is the preferred mode of transportation in Europe.

5.2 Impact of research results

A large part of the research conducted in the team is of fundamental level. Direct applications lie in numerical arts, cloth design, sports, and environmental studies, all of these being of limited negative impact for the environment. Collaborations with industry leading specially harmful activities to the environment are avoided.

6 Highlights of the year

6.1 Awards

Our 2011 paper entitled "A hybrid iterative solver for robustly capturing Coulomb friction in hair dynamics" [28] has been awarded a Test of Time Award at Siggraph Asia 2023 in Sydney.

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[2] In France this new trend was particularly stimulated by the work of Yves Pomeau, who convinced many young scientists to study the nonlinear physics of common objects such as paper, plants, or hair [36].

[3] It is however amusing to observe that research in these areas is quite successful in obtaining the IG Nobel prize [16, 40], thus still being considered as an exotic research topic by physicists.

This paper introduced the first nonsmooth algorithm able to capture dry friction in flexible fibre assemblies in a fast and robust manner. The method was inspired by the French Moreau school on nonsmooth mechanics pioneered in the 80's, which was essentially applied to granular materials and rigid bodies at that time. The frictional contact algorithm was implemented in the free so-bogus library and was used in several FX movies (including *The Hobbit: An unexpected journey* and *Superman: Man of steel*) for simulating hair and fur realistically.

7   New software, platforms, open data

7.1   New software

7.1.1   Feel++

Keywords: High order finite elements, Discontinuous Galerkin, High-Performance Computing

Functional Description: Feel++ is a high-performance C++ library for the resolution of general variational formulations, including continuous and discontinuous Galerkin methods, finite element or spectral element methods, reduced basis formulations, etc. It features a high-level domain specific embedded language (DSEL) for Galerkin methods, space dimension-agnostic computation kernels and seamless and automatic parallelism. It also includes applicative toolboxes to solve physics problems in fluid mechanics, solid mechanics, thermal conduction, and the corresponding multi-physics coupling.

Release Contributions: - Support of distance-based contact forces between immersed bodies - BVH implementation for contact pruning - Various improvements in expression support

URL: https://docs.feelpp.org/home/index.html

Contact: Thibaut Metivet

Partners: Université de Strasbourg, UGA, Inria

7.1.2   Sand6

Keywords: Granular matter, Frictional contact, Drücker-Prager rheology

Scientific Description: sand6 is a software to simulate the dynamics of granular matter using a continuum approach accounting for non-smooth flow rules. It is based on the nonsmooth Material Point Method described in [DBD16a] and is currently maintained and developed in the team for various aspects related to the modeling of frictional contact in continuous systems.

Functional Description: Simulation of granular matter as a continuum media

Release Contributions: This software contains a C++ implementation of the algorithms described in the 2016 ACM SIGGRAPH paper entitled "A Semi-Implicit Material Point Method for the Continuum Simulation of Granular Materials" by Gilles Daviet and Florence Bertails-Descoubes. It is currently maintained and further developed by Thibaut Métivet. It was notably updated and physically validated in the context of transient granular collapses in the paper [Rousseau G, Métivet T, Rousseau H, Daviet G, Bertails-Descoubes F. Revisiting the role of friction coefficients in granular collapses: confrontation of 3-D non-smooth simulations with experiments. Journal of Fluid Mechanics. 2023 Nov,975:A14.].

URL: https://gitlab.inria.fr/elan-public-code/sand6

Publications: hal-01310189, hal-03845323

Contact: Thibaut Metivet

Participants: Thibaut Metivet, Florence Bertails Descoubes, Gilles Daviet

7.1.3   MERCI

Name: Energy Minimisation of Curvature-based numerical models for Inextensible Ribbons

Keywords: Thin elastic rod, Thin elastic ribbon, Physical simulation

Scientific Description: MERCI is a C++/lua software for computing the statics of thin elastic ribbons discretised with curvature-based elements. It is based on the super-ribbon model described in [Charrondière et al. 2020, Charrondière et al. 2024], and relies on the free [IPOPT](https://coin-or.github.io/Ipopt/) optimisation software (coinor project) for the static solver. The ribbon can be clamped at one or both ends, and even closed. Contact is treated by contraints with planes. Once the setup is defined, the equilibrium of the ribbon under the specified boundary conditions, external forces, and constraints, is computed. MERCI can be used as a C++ library, or via its lua interface.

Reference code of the PhD thesis:

Raphaël Charrondière, "Modélisation numérique de rubans par éléments en courbures", 2021, Université Grenoble Alpes, https://hal.inria.fr/tel-03545017v2

and of the following papers:

R. Charrondière, F. Bertails-Descoubes, S. Neukirch, V. Romero, "Numerical modeling of inextensible elastic ribbons with curvature-based elements", Computer Methods in Applied Mechanics and Engineering 364, June 2020, p. 1–32, [doi:10.1016/j.cma.2020.112922], [hal-02515877].

R. Charrondière, S. Neukirch, F. Bertails-Descoubes, "MERCI: Mixed curvature-based elements for computing equilibria of thin elastic ribbons", to appear in 2024.

Functional Description: MERCI is a C++/lua software for computing the statics of thin elastic ribbons discretised with curvature-based elements. It is based on the super-ribbon model described in [Charrondière et al. 2020, Charrondière et al. 2024], and relies on the free [IPOPT](https://coin-or.github.io/Ipopt/) optimisation software (coinor project) for the static solver. The ribbon can be clamped at one or both ends, and even closed. Contact is treated by contraints with planes. Once the setup is defined, the equilibrium of the ribbon under the specified boundary conditions, external forces, and constraints, is computed. MERCI can be used as a C++ library, or via its lua interface.

Reference code of the PhD thesis:

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and of the following papers:

R. Charrondière, F. Bertails-Descoubes, S. Neukirch, V. Romero, "Numerical modeling of inextensible elastic ribbons with curvature-based elements", Computer Methods in Applied Mechanics and Engineering 364, June 2020, p. 1–32, [doi:10.1016/j.cma.2020.112922], [hal-02515877].

R. Charrondière, S. Neukirch, F. Bertails-Descoubes, "MERCI: Mixed curvature-based elements for computing equilibria of thin elastic ribbons", to appear in 2024.

Publication: hal-02515877

Authors: Raphael Charrondiere, Raphael Charrondiere, Raphael Charrondiere, Florence Descoubes, Sébastien Neukirch

Contact: Florence Bertails Descoubes

Partners: Sorbonne Université, UGA

7.1.4 ElanFab

Keywords: Experimental mechanics, Experimental design, Thin elastic ribbon, Thin elastic rod, Thin elastic shell, Frictional contact

Functional Description: Experimental platform of the ELAN team.

The aim of this platform is to experimentally explore the mechanics and geometry of highly deformable elastic objects of low dimensions (rods, fibers, plates, shells).

The platform allow us to fabricate with controlled materials and geometries elastic objects. By means of state of the art manufacturing techniques we are able to make curved elastic objects, with a controlled target geometry. For the moment we use elastomeric materials to remain in the elastic regime, however we are interested in exploring new materials to include viscous and plastic effects.

Our platform has a modular mechanical testing device that allow load and tensile testing in multiple configurations for a wide range of force magnitudes, from 1e-3 to 100 Newtons. In this setup we have tested highly compliant, as well as, very stiff materials, for example we study the tensile response of feathers and elastic knotted rods.

The platform is constantly undergoing new improvements to allow us to obtain geometrical information by means of a combination of image analysis and computer vision techniques. Multiple views are obtained by using multiple cameras and mirrors or by using one single camera that moves in a highly controlled manner. Furthermore we are implementing the use of a semi fast camera to study dynamic phenomena of complex elastic objects assemblies. We are also implementing structured light into our setup to improve the accuracy of our measurements.

Author: Victor Romero Gramegna

Contact: Victor Romero Gramegna

7.1.5 so-bogus

Keywords: Frictional contact, Constraint solving

Scientific Description: The so-bogus library, based on the bogus library, implements several methods (analytical solver, Gauss-Seidel solver, root-finding solver, etc.) for solving Signorini-Coulomb problems in 2D and 3D. It serves as the reference code for the paper "A Hybrid Iterative Solver for Robustly Capturing Coulomb Friction in Hair Dynamics", Daviet et al. 2011, ACM Transactions on Graphics (SIGGRAPH Asia 2011).

Functional Description: A fast and robust solver for capturing frictional contact between many Lagrangian systems with exact Coulomb friction. Reference code for the paper "A Hybrid Iterative Solver for Robustly Capturing Coulomb Friction in Hair Dynamics", Daviet et al. 2011, ACM Transactions on Graphics (SIGGRAPH Asia 2011).

The so-bogus software is currently maintained and further developed by Thibaut Métivet and Florence Bertails-Descoubes.

URL: https://gitlab.inria.fr/elan-public-code/so-bogus

Publication: hal-00647497

Contact: Thibaut Metivet

Participants: Florence Descoubes, Thibaut Metivet, Gilles Daviet

7.1.6 meche2D

Name: meche2D

Keywords: Physical simulation, Thin elastic rod, Frictional contact

Scientific Description: Meche2D is a library for coupling 2D elastic fibers with frictional contact. It relies on the supercircle code for fibers, and on so-bogus for frictional contact.

Functional Description: Meche2D is a library for coupling 2D elastic fibers with frictional contact. It relies on the supercircle code for fibers, and on so-bogus for frictional contact. New since 2020-2023: fibers can be clamped or unclamped, and contact detection is exact between two circular arcs.

Publication: https://hal.science/hal-03996830v2

Contact: Florence Bertails Descoubes

Participants: Florence Bertails Descoubes, Emile Hohnadel, Thibaut Metivet

7.2 New platforms

7.2.1 Elanfab 7.1.4

Participants: Victor Romero.

The aim of this platform is to experimentally explore the mechanics and geometry of highly deformable elastic objects of low dimensions (rods, fibers, plates, shells).

The platform allow us to fabricate with controlled materials and geometries elastic objects. By means of state of the art manufacturing techniques we are able to make curved elastic objects, with a controlled target geometry. For the moment we use elastomeric materials to remain in the elastic regime, however we are interested in exploring new materials to include viscous and plastic effects.

Our platform has a modular mechanical testing device that allows load and tensile testing in multiple configurations for a wide range of force magnitudes, from 1e-3 to 100 Newtons. In this setup we have tested highly compliant, as well as, very stiff materials, for example we study the tensile response of feathers 8.6 and elastic knotted rods 8.7.

The platform is constantly undergoing new improvements to allow us to obtain geometrical information by means of a combination of image analysis and computer vision techniques. Multiple views are obtained by using multiple cameras and mirrors or by using one single camera that moves in a highly controlled manner. Furthermore we are implementing the use of a semi-fast camera to study dynamic phenomena of complex elastic objects assemblies. We are also implementing structured light into our setup to improve the accuracy of our measurements.

8 New results

8.1 Nonsmooth simulations of 3D Drucker-Prager granular flows and validation against experimental column collapses

Participants: Thibaut Métivet, Florence Bertails-Descoubes.

In collaboration with Gauthier Rousseau (TU Wien, formerly post-doc in the team), Hugo Rousseau (INRAE) and Gilles Daviet (NVIDIA, formerly Ph.D. student in the team), we have performed thorough comparisons between the predictions of our numerical solver **Sand6** for granular flows 7.1.2, and collapse experiments conducted in a narrow channel (in collaboration with EPFL). We have shown that our

nonsmooth simulator, which relies on a constant friction coefficient corresponding to the yield angle of a granular heap, is able to reproduce with high fidelity various experimental granular collapses over inclined erodible beds. Our results, obtained for two different granular materials and for various bed inclinations, suggest that a simple constant friction rheology choice remains reasonable for capturing a large variety of unsteady granular flows at low inertial number. Using the versatility of our numerical approach, we have further analysed the possible biases pertaining to laboratory-scale experiments, and shown that, in the case of granular collapses, accurate predictions could be performed as long as care was taken in measuring yield angle of the granular material appropriately. This study has been published in the Journal of Fluid Mechanics [8], and all the research data provided for reproducibility at doi.org/10.5281/zenodo.7288829.

8.2 Randomly stacked open-cylindrical shells as a functional mechanical device

Participants: Émile Hohnadel, Thibaut Métivet, Florence Bertails-Descoubes.

In collaboration with Tomohiko Sano (Keio University), we have studied the mechanical behaviour of open cylindrical shells, randomly stacked in 2D configurations. Using both numerical simulations (relying on our validated curvature-based fibre model with non-smooth frictional contact [7]) and experiments, we have shown that despite the randomness of the configurations, the stacked shells exhibit robust macroscopic dissipative properties, involving complex interplay between elasticity and friction, which control the occurrence of snap-fit events at the micro-scale. Our results demonstrate that the rearrangement of flexible components could yield versatile but predictive mechanical responses, paving the way to new kinds of metamaterials. These results have been presented at the HFSS 2023 international conference [12] and published in Nature Communications Materials [9].

8.3 Hydrodynamic model for fish locomotion

Participants: Thibaut Métivet.

In collaboration with Bruno Ventéjou (co-supervised post-doc) and Philippe Peyla (LIPhy, UGA), we study the respective roles of hydrodynamic and social interactions within schools of fish, in the context of the FISHSIF ANR project. As a first step toward the simulation of large assemblies of swimming fish, we have developed a simplified hydrodynamic model, able to account for individual fish swimming and stigmergy, in particular regarding the generation of vortices wakes, without the need to introduce deformation of the body of the fish. The scaling analysis of our model falls accurately within universal swimming laws observed among many aquatic species [38], and further extends the laws of swimming efficiency to the low-inertial regime. Our model has been presented at the 2023 APS Division of Fluid Dynamics international conference [14], and a publication is in preparation.

8.4 Mixed elements for a curvature-based ribbon model

Participants: Florence Bertails-Descoubes.

In collaboration with Sébastien Neukirch (Institut D'Alembert, Sorbonne Université) and our former Ph.D. student Raphaël Charrondière, we have improved our initial curvature-based numerical model for thin elastic ribbons [2] by developing a mixed strategy, where each ribbon element is treated independently, and glued to each other only at the final solving stage through well-chosen bilateral constraints.

Thanks to this mixed variational strategy, which yields a banded Hessian, our algorithm recovers the linear complexity of low-order models while preserving the quadratic convergence of curvature-based models. As a result, our approach is scalable to a large number of elements, and suitable for

various boundary conditions and unilateral contact constraints, making it possible to handle challenging scenarios such as confined buckling experiments or Möbius bands with contact. Additionally, our numerical model can incorporate various ribbon energies, including the recent [17] model for quasi-developable ribbons recently introduced in Physics, which allows to transition smoothly between a rectangular Kirchhoff rod and a (developable) Sadowsky ribbon. Our numerical scheme is carefully validated against demanding experiments of the Physics literature, which demonstrates its accuracy, efficiency, robustness, and versatility.

This work is currently under review for a journal publication.

8.5 High-order contact detection between fibres

Participants:	Octave Crespel, Émile Hohnadel, Thibaut Métivet, Florence Bertails-Descoubes.

In this work we analyse the contact forces yielded by fibre models coupled to a segment-based contact detection algorithm. We note the occurrence of spurious jumps which, to the best of our knowledge, were never reported before. We demonstrate that these artifacts are actually directly linked to the low-order contact detection being used between the fibre and the obstacles, and that they worsen as curvature at contact increases. Low-order detection, based on segment proxys, is classically used due to its simplicity of treatment, even for fibre models possessing a higher-order geometry like super-helices. We show that spurious artifacts occur whatever the fibre model and contact response solver used, as soon as a segment-segment detection scheme is employed. To remove such numerical artifacts in the force profile, which can accumulate to yield large force errors, we introduce an efficient high-order detection algorithm, relying on an efficient adaptive pruning strategy.

This work, available as a preprint [15], is currently under review for a journal publication.

8.6 Numerical modeling of a feather's vane, a highly anisotropic membrane

Participants:	Jouve Jean, Romero Victor, Florence Bertails-Descoubes.

In the context of Jean Jouve's doctoral thesis, in collaboration with Rahul Narain (Indian Institute of Technology Delhi, India) and Theodore Kim (Yale University, United States), we have developed a macroscopic shell simulator capable of dealing with highly anisotropic materials, such as feathers.

Feathers have a multi-scale structure with very interesting properties. From a central stiff rod (the rachis) hundreds of small fibers (barbs) come out to form a membrane-like structure, which is held together by yet another set of smaller fibers (barbules). This intricate structure drives the elastic response of the membrane and gives feathers the capability of reversible rupture. Our interest is to produce a simulator capable of realistically portray the deformation of a feather. This simulation is challenging because the membrane has two main directions with elastic coefficients differing by 4 orders of magnitude. We have compared and validated our results with experimental data, and an article has been submitted for publication.

8.7 Untangling the physics of self-locking in tight knots

Participants:	Alexandre Teixeira Da Silva, Thibaut Métivet, Victor Romero, Florence Bertails-Descoubes.

In the context of the Ph.D. of Alexandre Teixeira Da Silva, co-supervised by Florence Bertails-Descoubes, Thibaut Métivet, Victor Romero, and Mélina Skouras (Anima, Inria GRA), we study the mechanical behaviour of tight knots, with a particular focus on the respective roles of elasticity and friction in the

self-locking events – knot configurations where the knot remains tight even in the absence of external charge. We investigate such behaviour using both numerical simulations, performed with the PolyFEM software, and experiments, conducted within our 7.1.4 platform. Preliminary results have been presented for the overhand knot configuration at the HFSS international conference [13], along with an important validation study of the PolyFEM numerical methods in the context of large-strains mechanics.

8.8 Frictional three-point bending test: disentangling the role of friction through real and numerical experiments

Participants: Émile Hohnadel, Thibaut Métivet, Florence Bertails-Descoubes.

In collaboration with Joël Marthelot, Ignacio Andrade-Silva and Olivier Pouliquen (IUSTI, Aix Marseille Université), we have investigated the role of friction in the well-known three-point bending test, traditionally used to measure the bending modulus of slender structures. By performing experiments and numerical simulations, both compared to our new theoretical model, we have devised an efficient protocol to disentangle the respective roles of elasticity and friction in the force response of an indented rod lying on frictional supports, thereby allowing accurate and independent measurements of both the bending modulus and the friction coefficient of the considered material. This work has been presented at the ESMC 2022 international conference [41], and an article is in preparation.

9   Partnerships and cooperations

9.1   International research visitors

9.1.1   Visits of international scientists

Rahul Narain

Status: Assistant Professor

Institution of origin: Indian Institute of Technology Delhi

Country: India

Dates: October 9 - December 22, 2023

Context of the visit : Scientific collaboration on our feather modelling project

Mobility program/type of mobility: Inria Invited Researcher Program

9.1.2   International collaborations

- Scientific collaboration with Tomohiko Sano (Sigma Lab, Japan) on the analysis of random stacks of open elastic rings
- Scientific collaboration with Rahul Narain (IIT Delhi, India) and Theodore Kim (Yale, United States) on the numerical modelling of feathers

9.2   European initiatives

9.2.1   H2020 projects

THREAD   THREAD project on cordis.europa.eu

Title: Joint Training on Numerical Modelling of Highly Flexible Structures for Industrial Applications

Duration: From October 1, 2019 to March 31, 2024

Partners:

- University of Liège (ULIEGE), Belgium
- Ecole Nationale Supérieure d'Arts et Métiers (ENSAM), France
- Friedrich-Alexander-Universitaet Erlangen-Nuernberg (FAU), Germany
- Martin-Luther-Universitat Halle-Wittenberg (MLU), Germany
- Indian Institute of Technology Delhi
- Fraunhofer Gesellschaft Zur Forderung der Angewandten Forschung ev (Fraunhofer), Germany
- Sveuciliste u Rijeci, Gardevinski Fakultet u Rijeci (University of Rijeka, Faculty of civil engineering Unirifce), Croatia
- C3M DOO, Center za Racunalnistvo Vmehaniki Kontinuuma - Modeliranje in Trzenje (C3M DOO), Slovenia
- Universitaet Innsbruck (UIBK), Austria
- Centrale-Supelec, France
- Norges Teknisk-Naturvitenskapelige Universitet NTNU (NTNU), Norway
- Univerza v Ljubjani (UL), Slovenia
- Universidad de Sevilla, Spain

Inria contact: Florence DESCOUBES

Coordinator:

Summary: Virtual prototyping is a cornerstone in modern product development cycles: It accelerates the design process, reduces costs and improves product performance and quality. Highly flexible slender structures like yarns, cables, hoses or ropes are essential parts of high-performance engineering systems. The complex response of such structures in real operational conditions is far beyond the capabilities of current virtual prototyping tools.

There is a pressing need for a new generation of young scientists capable of solving fundamental problems related to slender structures and transferring results to applications. THREAD addresses the mechanical modelling, mathematical formulations and numerical methods for highly flexible slender structures. It brings mechanical engineers and mathematicians together around major challenges in industrial applications and open-source simulation software development. It establishes an innovative modelling chain starting from detailed 3D modelling and experimental work to build validated 1D nonlinear rod models, which are then brought to a system-level simulation thanks to the outstanding numerical properties of the developed algorithms. This holistic approach combines advanced concepts in experimental and theoretical structural mechanics, non-smooth dynamics, computational geometry, discretisation methods and geometric numerical integration and will enable the next generation of virtual prototyping.

The ESRs will receive comprehensive local and network-wide training covering state-of-the-art research topics as well as valuable transferable skills. They will benefit from close cooperation with twelve industrial partner organisations implementing a comprehensive programme of research secondments and contributing their experience. As a main objective of THREAD, interdisciplinary and inter-sectoral training boosts the career development of young researchers and supports them to solve future challenges.

9.3   National initiatives
FISHSIF ANR Project

Participants: Thibaut Métivet.

Title: FISHSIF: Fish In Silico with Hydrodynamics and Social Forces

Duration: 01/10/2021 - 31/03/2025

Summary: The FISHSIF project has received a three-year funding from the ANR (Agence Nationale pour la Recherche). The goal of this project is to introduce dynamical cognition models within full hydrodynamic simulations in order to understand the role played by social or flow interactions in the organisation and behaviour of schools of fish. The project will be led in a collaboration between the ELAN team, the Laboratoire Interdisciplinaire de Physique (LIPhy, UGA/CNRS) and the Laboratoire de Psychologie et NeuroCognition (LPNC, UGA/CNRS).

Partners:
- ELAN Inria project-team
- Laboratoire Interdisciplinaire de Physique (LIPhy), Université Grenoble-Alpes (UGA)
- Laboratoire de Psychologie et NeuroCognition (LPNC), Université Grenoble-Alpes (UGA)

National collaborations

- Long-term collaboration with Arnaud Lazarus and Sébastien Neukirch (Institut Jean le Rond d'Alembert, Sorbonne Université).

- Long-term collaboration with Christophe Prud'homme and Vincent Chabannes (Université de Strasbourg and Centre de modélisation et de simulation de Strasbourg).

- Collaboration with Olivier Pouliquen and Joël Marthelot (IUSTI, Aix-Marseille University).

- Collaboration with Philippe Peyla, Aurélie Dupont (LIPhy, UGA/CNRS) and Christian Graff (LPNC, UGA/CNRS) within the FISHSIF project.

Regional collaborations

- Collaboration with Mélina Skouras (Inria/LJK - Anima team).

- Collaboration with Stefanie Wuhrer and Jean-Sébastien Franco (Inria/LJK - Morphéo team).

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10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: selection

Member of the conference program committees   Florence Bertails-Descoubes was member of the ACM Siggraph Technical Papers Program Committee in 2023.

10.1.2 Journal

Member of the editorial boards   Since 2021, Florence Bertails-Descoubes is Associate Editor of ACM Transactions on Graphics.

Reviewer - reviewing activities

- Siggraph (ACM, accepted papers published in Transactions on Graphics)

- Siggraph Asia (ACM, accepted papers published in Transactions on Graphics)

- Journal of the Mechanics and Physics of Solids (JMPS, Elsevier)

10.1.3   Invited talks

- Septembre 2023: Invited lecture [10] at the international conference on High Flexible Solid Structures (Florence Bertails-Descoubes)

10.1.4   Research administration

Ph.D. Award Committees

- Florence Bertails-Descoubes was member of the international ACM Siggraph Ph.D. award in 2023.

- Florence Bertails-Descoubes was member of the national GdR IG-RV Ph.D. award in 2023.

National Selection Committees   Florence Bertails-Descoubes was member of the 2023 Jury d'Admission for the national concours "Chargées and Chargés de Recherche de Classe Normale".

Local Selection Committees   Florence Bertails-Descoubes was member of the 2023 Maître de Conférence Jury "MCF 0194" at Grenoble INP and Université Grenoble Alpes, and of the 2023 Maître de Conférence Jury "MCF 0163" at Université Grenoble Alpes.

10.2   Teaching - Supervision - Juries

10.2.1   Teaching

Licence

- Émile Hohnadel: Langages et automates, 27h, L2, Université Grenoble Alpes

- Jean Jouve: Méthodes numériques de base, 26h, Ensimag 1A, Grenoble INP.

- Victor Romero: Electromagnétisme et optique pour la chimie, PHY405, 33h, DLST, Université Grenoble Alpes, Grenoble.

Master

- Thibaut Métivet, Florence Bertails-Descoubes and Jean Jouve: Mécanique Numérique, 36h, Ensimag 3A, Grenoble INP.

10.2.2   Supervision

Post-doctorate

- Bruno Ventéjou: 01/05/2022-, supervised by Thibaut Métivet and Philippe Peyla (LIPhy, UGA)

Ph.D.

- Alexandre Teixeira : 01/02/2021-, supervised by Thibaut Métivet, Florence Bertails-Descoubes and Mélina Skouras (Anima, Inria GRA)

- Émile Hohnadel : 01/09/2021-, supervised by Florence Bertails-Descoubes and Thibaut Métivet

- Jean Jouve : 01/09/2021-, supervised by Victor Romero and Florence Bertails-Descoubes

Internship

- Adrien Decosse : 25/05/2023-14/07/2023, supervised by Thibaut Métivet and Florence Bertails-Descoubes

10.2.3   Juries

Florence Bertails-Descoubes was member of the Ph.D committee of Ke Wu (as an examiner), March 31st 2023, at Centrale Lille, of Pierre Zins (as a president), April 25th 2023, at Inria Grenoble, and of Maxime Calka (as a president), June 8th 2023, at TIMC Grenoble.

10.3   Popularization

In October 2023, Octave Crespel participated in *La Fête de la Science* organised at Inria Grenoble.

11   Scientific production

11.1   Major publications

[1]   F. Bertails-Descoubes, A. Derouet-Jourdan, V. Romero and A. Lazarus. 'Inverse design of an isotropic suspended Kirchhoff rod: theoretical and numerical results on the uniqueness of the natural shape'. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 474.2212 (Apr. 2018), pp. 1–26. DOI: 10.1098/rspa.2017.0837. URL: https://hal.inria.fr/hal-01827887.

[2]   R. Charrondière, F. Bertails-Descoubes, S. Neukirch and V. Romero. 'Numerical modeling of inextensible elastic ribbons with curvature-based elements'. In: *Computer Methods in Applied Mechanics and Engineering* 364 (1st June 2020), pp. 1–32. DOI: 10.1016/j.cma.2020.112922. URL: https://hal.inria.fr/hal-02515877.

[3]   J. Li, G. Daviet, R. Narain, F. Bertails-Descoubes, M. Overby, G. Brown and L. Boissieux. 'An Implicit Frictional Contact Solver for Adaptive Cloth Simulation'. In: *ACM Transactions on Graphics*. Proceedings SIGGRAPH 2018 37.4 (Aug. 2018), pp. 1–15. DOI: 10.1145/3197517.3201308. URL: https://hal.inria.fr/hal-01834705.

[4]   M. Ly, R. Casati, F. Bertails-Descoubes, M. Skouras and L. Boissieux. 'Inverse Elastic Shell Design with Contact and Friction'. In: *ACM Transactions on Graphics* 37.6 (Nov. 2018), pp. 1–16. DOI: 10.1145/3272127.3275036. URL: https://hal.inria.fr/hal-01883655.

[5]   T. Metivet, A. Sengers, M. Ismail and E. Maitre. 'Diffusion-redistanciation schemes for 2D and 3D constrained Willmore flow: application to the equilibrium shapes of vesicles'. In: *Journal of Computational Physics* 436 (2021), p. 110288. DOI: 10.1016/j.jcp.2021.110288. URL: https://hal.archives-ouvertes.fr/hal-02905870.

[6]   A.-H. Rasheed, V. Romero, F. Bertails-Descoubes, S. Wuhrer, J.-S. Franco and A. Lazarus. 'A Visual Approach to Measure Cloth-Body and Cloth-Cloth Friction'. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* (July 2021). DOI: 10.1109/TPAMI.2021.3097547. URL: https://hal.inria.fr/hal-03285624.

[7]   V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch and F. Bertails-Descoubes. 'Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact'. In: *ACM Transactions on Graphics* 40.4 (1st Aug. 2021), Article 66: 1–19. DOI: 10.1145/3450626.3459931. URL: https://hal.inria.fr/hal-03217459.

11.2   Publications of the year

International journals

[8]   G. Rousseau, T. Métivet, H. Rousseau, G. Daviet and F. Bertails-Descoubes. 'Revisiting the role of friction coefficients in granular collapses: confrontation of 3-D non-smooth simulations with experiments'. In: *Journal of Fluid Mechanics* (15th Nov. 2023). DOI: 10.1017/jfm.2023.835. URL: https://inria.hal.science/hal-03845323.

[9]   T. G. Sano, E. Hohnadel, T. Kawata, T. Métivet and F. Bertails-Descoubes. 'Randomly stacked open cylindrical shells as functional mechanical energy absorber'. In: *Communications Materials* 4.59 (25th Aug. 2023). DOI: `10.1038/s43246-023-00383-2`. URL: `https://hal.science/hal-0399 6830`.

International peer-reviewed conferences

[10]   F. Bertails-Descoubes. 'Simulating fibre assemblies: from Hollywood illusions to physical predictions'. In: HFSS 2023 – International Conference on Highly Flexible Slender Structures. Rijeka, Croatia, 2023, pp. 1–3. URL: `https://inria.hal.science/hal-04265998`.

[11]   E. Hohnadel, O. Crespel, T. Métivet and F. Bertails-Descoubes. 'Accurate contact detection and response in fibre assemblies with friction'. In: HFSS 2023 – International Conference on Highly Flexible Slender Structures. Rijeka, Croatia, 2023, pp. 1–1. URL: `https://hal.science/hal-043 88542`.

[12]   T. Métivet, E. Hohnadel, T. G. Sano, T. Kawata and F. Bertails-Descoubes. 'Investigating the compaction of open ring stacks through real and numerical experiments'. In: HFSS 2023 - International Conference on Highly Flexible Slender Structures. Rijeka, Croatia, 2023, pp. 1–2. URL: `https://ha l.science/hal-04225083`.

[13]   A. Teixeira da Silva, T. Métivet, V. Romero, M. Skouras and F. Bertails-Descoubes. 'Untangling the physics of self-locking in tight knots'. In: HFSS 2023 - International Conference on Highly Flexible Slender Structures. Rijeka, Croatia, 2023, pp. 1–2. URL: `https://inria.hal.science/hal-0438 8669`.

Conferences without proceedings

[14]   B. Ventéjou, T. Métivet, A. Dupont, C. Graff and P. Peyla. 'A hydrodynamic toy model for fish locomotion'. In: DFD 2023 - 76th Annual Meeting of the Division of Fluid Dynamics. Washinghton DC, United States, 19th Nov. 2023. URL: `https://hal.science/hal-04254582`.

Reports & preprints

[15]   O. Crespel, E. Hohnadel, T. Métivet and F. Bertails-Descoubes. *Contact detection between curved fibres: high order makes a difference*. 27th Dec. 2023. URL: `https://hal.science/hal-04364565`.

11.3   Cited publications

[16]   B. Audoly and S. Neukirch. 'Fragmentation of Rods by Cascading Cracks: Why Spaghetti Does Not Break in Half'. In: *Physical Review Letters* 95.9 (2005), p. 095505.

[17]   B. Audoly and S. Neukirch. 'A one-dimensional model for elastic ribbons: A little stretching makes a big difference'. In: *Journal of the Mechanics and Physics of Solids* 153 (2021), p. 104457. DOI: `10.1016/j.jmps.2021.104457`.

[18]   D. Baraff. 'Analytical methods for dynamic simulation of non-penetrating rigid bodies'. In: *Computer Graphics Proceedings (Proc. ACM SIGGRAPH'89)*. New York, NY, USA: ACM, 1989, pp. 223–232.

[19]   F. Bertails. 'Linear Time Super-Helices'. In: *Computer Graphics Forum (Proc. Eurographics'09)* 28.2 (Apr. 2009). URL: `http://www-ljk.imag.fr/Publications/Basilic/com.lmc.publi .PUBLI_Article@1203901df78_1d3cdaa/`.

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        curvatures'. In: *The 4th Joint International Conference on Multibody System Dynamics*. Montréal,
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