RESEARCH CENTRE

Inria Centre
at Université Côte d’Azur

IN PARTNERSHIP WITH:
CNRS, Université Côte d’Azur

2023
ACTIVITY REPORT

Project-Team
CASTOR

Control, Analysis and Simulations for TOkamak Research

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Digital Health, Biology and Earth

THEME

Earth, Environmental and Energy Sciences

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Creation of the Project-Team: 2014 July 01

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B4. – Energy

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1 Team members, visitors, external collaborators

Research Scientists

- Hervé Guillard [Team leader, INRIA, Senior Researcher, HDR]
- Blaise Faugeras [CNRS, Researcher, HDR]
- Florence Marcotte [INRIA, Researcher]

Faculty Members

- Didier Auroux [UNIV COTE D'AZUR, Associate Professor, HDR]
- Jacques Blum [UNIV COTE D'AZUR, Emeritus, HDR]
- Cédric Boulbe [UNIV COTE D'AZUR, Associate Professor]
- Boniface Nkonga [UNIV COTE D'AZUR, Professor, HDR]
- Francesca Rapetti-Gabellini [UNIV COTE D'AZUR, Associate Professor, HDR]
- Afeintou Sangam [UNIV COTE D'AZUR, Associate Professor]

Post-Doctoral Fellows

- Ashish Bhole [CNRS, Post-Doctoral Fellow, until Nov 2023]
- Alexandre Vieira [CNRS, Post-Doctoral Fellow]

PhD Students

- Guillaume Gros [UNIV COTE D'AZUR]
- Louis Lamerand [UNIV COTE d'AZUR]

Technical Staff

- Jeaniffer Vides Higueros [LEMMA Company, Engineer]

Interns and Apprentices

- Salma Janati [INRIA, Intern, from May 2023 until Jun 2023]

Administrative Assistant

- Nathalie Nordmann [INRIA, from Apr 2023]

Visiting Scientists

- Praveen Chandrashekarappa [TIFR INDE, from Jun 2023 until Aug 2023]
- Raphael Onguene [Douala IUT , from Sep 2023 until Sep 2023]

2 Overall objectives

In order to fulfill the increasing demand, alternative energy sources have to be developed. Indeed, the current rate of fossil fuel usage and its serious adverse environmental impacts (pollution, greenhouse gas emissions, ...) lead to an energy crisis accompanied by potentially disastrous global climate changes.
Controlled fusion power is one of the most promising alternatives to the use of fossil resources, potentially with a unlimited source of fuel. France with the ITER and Laser Megajoule facilities is strongly involved in the development of these two parallel approaches to master fusion that are magnetic and inertial confinement. Although the principles of fusion reaction are well understood from nearly sixty years, (the design of tokamak dates back from studies done in the '50 by Igor Tamm and Andreï Sakharov in the former Soviet Union), the route to an industrial reactor is still long and the application of controlled fusion for energy production is beyond our present knowledge of related physical processes. In magnetic confinement, beside technological constraints involving for instance the design of plasma-facing component, one of the main difficulties in the building of a controlled fusion reactor is the poor confinement time reached so far. This confinement time is actually governed by turbulent transport that therefore determines the performance of fusion plasmas. The prediction of the level of turbulent transport in large machines such as ITER is therefore of paramount importance for the success of the researches on controlled magnetic fusion.
The other route for fusion plasma is inertial confinement. In this latter case, large scale hydrodynamical instabilities prevent a sufficiently large energy deposit and lower the return of the target. Therefore, for both magnetic and inertial confinement technologies, the success of the projects is deeply linked to the theoretical understanding of plasma turbulence and flow instabilities as well as to mathematical and numerical improvements enabling the development of predictive simulation tools.

CASTOR gathers the activities in numerical simulation of fusion plasmas with the activities in control and optimization done in the laboratory Jean-Alexandre Dieudonné of Université Côte d'Azur. The main objective of the CASTOR team is to contribute to the development of innovative numerical tools to improve the computer simulations of complex turbulent or unstable flows in plasma physics and to develop methods allowing the real-time control of these flows or the optimization of scenarios of plasma discharges in tokamaks. CASTOR is a common project between Inria, Université Côte d'Azur and CNRS through the laboratory Jean-Alexandre Dieudonné, UMR UNS-CNRS 7351, LJAD.

3 Research program

3.1 Plasma Physics

The main reseach topics are:

1. Modeling and analysis
   - Fluid closure in plasma
   - Turbulence
   - Plasma anisotropy type instabilities
   - Free boundary equilibrium (FBE)
   - Coupling FBE – Transport
   - MHD instabilities
2. Numerical methods and simulations
   - High order methods
   - Curvilinear coordinate systems
   - Equilibrium simulation
   - Anisotropy

- Solving methods and parallelism

3. Identification and control
   - Inverse problem: Equilibrium reconstruction
   - Open loop control
   - Dynamo effects in plasmas

4 Application domains

4.1 MHD and plasma stability in tokamaks

Participants: Hervé Guillard, Boniface Nkonga, Afeintou Sangam, Ali Elarif, Ashish Bhole.

The magnetic equilibrium in tokamaks results from a balance between the Lorentz force and the pressure gradient. Using Ampère law, a convenient description of this equilibrium is provided by the Grad-Shafranov equation. Of course, the magnetic equilibrium solution of the Grad-Shafranov equation is required to be stable. Actually any loss of MHD (Magneto-Hydro-Dynamics) stability can lead to the end of the existence of the plasma, the so-called disruptions that can affect negatively the integrity of the machine. The primary goal of MHD studies is therefore to determine the stability domain that constraints the operational range of the machine.
A secondary goal of MHD studies is to evaluate the consequences of possible disruptions in term of heat loads and stresses on the plasma facing components. In modern machines in the so-called H-mode some mild instabilities leading to a near oscillatory behavior are also known to exist. In particular, the so-called ELMs (Edge Localized Modes) are of particular importance since they can have large effects on the plasma facing components. The control and understanding of these instabilities is therefore of crucial importance for the design of future machines as ITER. Unfortunately, ELMs occur in the edge plasma and their modeling requires to take in account not only the intricate magnetic topology of this region where both open and closed field lines co-exist but also the existence of molecular and atomic processes involving neutrals.

At present, the linear theory of MHD stability is relatively well understood. However, the description of the non-linear behavior is far from being complete. As a consequence and due to the intrinsic difficulty of the subject, only a few numerical codes worldwide have been developed and validated for non linear MHD in tokamaks. One of these codes is the JOREK code developped since 2006 from a collaborative work between CEA-Cadarache (main developer), LABRI Bordeaux, LJAD-UniCA and Inria. A comprehensive description of JOREK is given in [19]

4.2 Long term plasma evolution and optimization of scenarii

Participants: Didier Auroux, Jacques Blum, Cédric Boulbe, Blaise Faugeras, Hervé Guillard.

The magnetic equilibrium evolves in time due to diffusion processes on the slow resistive diffusive time scale and moreover it has to be monitored with active and passive control based on external coils, current drive, heating system, particle or pellets injections. This set of control mechanism has to be modeled and this is the goal of real time codes or global evolution codes.
In the same order of ideas, the steering and control of the plasma from the beginning to the end of the discharge require the research of optimal trajectories through the space of operational parameters. This is usually performed in an empirical way in present Tokamaks, but the complexity of the problem requires today the use of optimization techniques for processes governed by MHD and diffusion-type equations.

4.3 Turbulence and models for the edge region of tokamaks

Participants: Didier Auroux, Louis Lamerand, Francesca Rapetti.

The edge region of the plasma is characterized by low temperature and density leading to an increase of the collision frequency that makes the edge plasma nearly collisional. This combined with the intricate magnetic topology of this region makes the development of kinetic codes adapted to the edge regions a real long term adventure. Consequently the fluid approach remains a standard one to study edge plasma turbulence. The use of optimal control theory to derive simplified models matching data either experimental or derived from direct numerical simulations is part of the objectives of the team.

4.4 High order accuracy methods

Participants: Blaise Faugeras, Herve Guillard, Boniface Nkonga, Francesca Rapetti.

We analyze the accuracy and robustness of C1 Finite Element (FE) for plasma equilibrium computations in presence of strongly anisotropic phenomena. Aligned Hermite Bezier (HB) FEs and non-aligned reduced Hiesh-Clough-Tocher (rHCT) FEs are coupled by the mortar element method for composite meshes.

Participants: Ana Alonso Rodriguez, Francesca Rapetti.

We have shown that high order polynomial interpolation with Nédélec edge elements can suffer from a Runge phenomenon similar to that well known for high order polynomial interpolation with Lagrange nodal elements. We have also studied a geometric approach for constructing physical degrees of freedom for sequences of finite element spaces of Nédélec type (first and second families).

4.5 Understanding magnetogenesis in stellar systems

Participants: Didier Auroux, Florence Marcotte.

The considerable diversity of long-lived magnetic fields observed in the Universe raises fundamental questions regarding their origin. Although it is now widely accepted that such fields are sustained by a dynamo instability in the electrically conducting fluid layers of astrophysical bodies, in most cases the very nature of the flow motions powering the dynamo is essentially unknown, and the conditions required for amplifying large-scale magnetic fields in non-convective stellar systems are poorly understood. We claim that optimal control represents a powerful tool to investigate the nonlinear stability of fully 3D, unsteady magnetohydrodynamic flows with respect to the dynamo instability. Nonlinear optimization can be also used as a physical diagnostic to gain novel understanding of the mechanisms that are most favorable to dynamo action in a natural system.

5 Social and environmental responsibility

5.1 Impact of research results

The main objective of the CASTOR team is to contribute to the development of the numerical tools used for the simulation of fusion plasma. Since the design of the next generation of fusion reactors relies on numerical simulation, the works done in CASTOR contribute to the search of a clean and decarbonated energy.

6 Highlights of the year

6.1 Awards

Florence Marcotte has been awarded a European ERC Starting Grant 'CIRCE' (2024-2028) to study the dynamo effect in stellar systems.

7 New results

7.1 Tokamak free-boundary plasma equilibrium computations in presence of non-linear materials

Participants: Blaise Faugeras, Cédric Boulbe, Guillaume Gros, Francesca Rapetti-Gabellini.

We consider the axisymmetric formulation of the equilibrium problem for a hot plasma in a tokamak. We adopt a non-overlapping mortar element approach, that couples piece-wise linear Lagrange finite elements in a region that does not contain the plasma and piece-wise cubic reduced Hsieh-Clough-Tocher finite elements elsewhere, to approximate the magnetic flux field on a triangular mesh of the poloidal tokamak section. The inclusion of ferromagnetic parts is simplified by assuming that they fit within the axisymmetric modeling and a new formulation of the Newton algorithm for the problem solution is stated, both in the static and quasi-static evolution cases. [8]

7.2 Numerical simulation of Tokamak plasma equilibrium evolution

Participants: Blaise Faugeras, Cédric Boulbe, Guillaume Gros, Francesca Rapetti-Gabellini.

This contribution focuses on the numerical methods recently developed in order to simulate the time evolution of a Tokamak plasma equilibrium at the resistive diffusion time scale. We develop on the method proposed by Heumann for the coupling of magnetic equilibrium and current diffusion. We introduce a new space discretization for the poloidal flux using C0 and C1 finite elements. This, together with the use of spline functions to represent the diamagnetic function in the resistive diffusion equation, enables to restrain numerical oscillations which can occur with the original method. We add to the model an evolution equation for electron temperature in the plasma. This enables us to compute consistently the plasma resistivity and the non-inductive current terms called bootstrap current needed in the resistive diffusion equation. It also enables us to evolve the pressure term in the simulation. These numerical methods are implemented in the plasma equilibrium code NICE. The code is coupled with a magnetic feedback controller through the MUSCLE3 library. This enables to simulate a prescribed plasma scenario. The results for an X-point formation scenario in the WEST tokamak are presented as a first illustration of the efficiency of the developed numerical methods.

7.3 Magnetic control of WEST plasmas through deep reinforcement learning

Participants: S. Kerboua-Benlarbi, R. Nouailletas, Blaise Faugeras, Eric Nardon *(IRFM,CEA)*, P. Moreau *(IRFM,CEA)*.

Tokamaks require magnetic control across a wide range of plasma scenarios. The coupled behavior of plasma dynamics makes deep learning a suitable candidate for efficient control in order to fulfil these high-dimensional and non-linear situations. For example, on the TCV tokamak, deep reinforcement learning has already been used for tracking of the plasma's magnetic equilibrium [1]. In this work, we apply such methods to the WEST tokamak, to address control of the plasma's shape, position, and current, in several relevant configurations. To this end, we developed a distributed framework to train an actor-critic agent on a C++ free boundary equilibrium code called NICE, in which resistive diffusion allows a more representative evolution of current profile throughout the simulation. The interface between components was done through UDS protocols for fast, asynchronous and reliable communication. The implemented tool handles feedback control of quantities of interest, with results showing flexibility of the method regarding the use of different training environments [17].

7.4 Turbulence and models for the edge region of tokamaks

Participants: Didier Auroux, Louis Lamerand, Francesca Rapetti-Gabellini.

The high-dimensional and multiscale nature of fusion plasma flows requires the development of reduced models to be implemented in numerical codes capable of capturing the main features of turbulent transport in a sufficiently short time to be useful during tokamak operation. This paper goes further in the analysis of the dynamics of the k-epsilon model based on the turbulent kinetic energy k and its dissipation rate epsilon [Baschetti et al., Nuc. Fus 61, 106020 (2021)] to improve the predictability of the transverse turbulent transport in simulation codes. Present 1D results show further capabilities with respect to current models (based on constant effective perpendicular diffusion) and on the standard quasi-linear approach. The nonlinear dependence of D in k and epsilon estimated from two additional transport equations allow to introduce some non-locality in the transport model. This is illustrated by the existence of parameter ranges with turbulence spreading. The paper also addresses another issue related to the uncertainties on the inherent free parameters of such reduced model. The study proposes a new approach in the fusion community based on a variational data assimilation involving the minimization of a cost function defined as the distance between the reference data and the calculated values. The results are good, and show the ability of the data assimilation to reduce uncertainties on the free parameters, which remains a critical point to ensure the total reliability of such an approach.

7.5 Coupling Hermite-Bezier quadrangular elements and Clough Tocher triangular elements

Participants: Ashish Bhole, Boniface Nkonga, Hervé Guillard, Francesca Rapetti-Gabellini.

The Jorek code for MHD studies uses fourth order C1 quadrangular finite elements. This leads to results with high spatial accuracy. However with this type of elements, the description of the boundary is difficult and moreover with polar meshes, geometrical singularities appear on X-points and magnetic axis. By relying on composite meshes, we have studied the coupling of the Hermite-Bezier quadrangular elements used in Jorek and of Clough Tocher triangular elements for an easier representation of the boundary and treatment of the mesh singularities. First results for the Poisson model problem with Dirichlet boundary conditions can be found in [6].

7.6 High-order Whitney finite elements in electromagnetics modeling

Participants: Francesca Rapetti-Gabellini, Ana Alonso Rodriguez (Univ. di Trento, Italy).

It is well known that Lagrange interpolation based on equispaced nodes can yield poor results. Oscillations may appear when using high degree polynomials. For functions of one variable, the most celebrated example has been provided by Carl Runge in 1901, who showed that higher degrees do not always improve interpolation accuracy. His example was then extended to multivariate calculus and in this work we show that it is meaningful, in an appropriate sense, also for Whitney edge elements, namely for differential 1-forms, as explained in [5].

We study a geometric approach for constructing physical degrees of freedom for sequences of finite element spaces. Within the framework of finite element systems, we propose new degrees of freedom for the second family of high order Nédélec finite elements in [12]

7.7 MHD model applied to massive material injections

Participants: Ashish Bhole, Boniface Nkonga, José Costa, Guido Huijsmans, Stanislas Pamela, Matthias Hoelzl.

Massive material injection (MMI) experiments in tokamaks aim to inject neutral gases (such as deuterium, neon, argon, etc.), also called impurities, into the tokamak plasma, giving rise to complex gas-plasma interactions. The atomic reactions during the interactions produce charged ions at different ionization levels. Multi-fluid MHD equations are appropriate candidates for gas-plasma interactions, where each fluid is characterized by its ionization level. In a recent work, under the assumption of coronal equilibrium, single fluid impurity transport modeling was proposed for the gas-plasma interactions, which provided satisfactory results for MMI simulations with the reduced MHD models. We have used this single fluid modeling in the single-temperature full MHD model context to obtain significant results. To get to this point, we had to face three critical challenges. First, the Galerkin FEMs give central approximations to the differential operators. Their use in the simulation of the convection-dominated flows may lead to dispersion errors, yielding entirely wrong numerical solutions. Second, high-order, high-resolution numerical methods produce high wave-number oscillations near shocks/discontinuities that adversely affect the numerical stability. Third, the aligned helpful mesh in this context of high anisotropy had drawbacks at critical points of the magnetic field. Then, we propose a numerical treatment for the geometric singularity at the polar grid center associated with a SUPG-like numerical stabilization. The stabilization strategy aims to identify the contributions of the modeling that need smoothing and apply it locally in space according to fitting criteria. The result is a stabilized bi-cubic Hermite Bézier FEM in the computational framework of the nonlinear magnetohydrodynamics (MHD) code JOREK.

A collisional-radiative treatment for impurities using coronal equilibrium assumption was implemented, benchmarked, and applied to validate simulations of shattered pellet injection (SPI) in the JET tokamak. Deuterium and impurity/mixed SPI simulations for the JET tokamak reproduce critical experimental observations, e.g., regarding radiation, showing that plasmoid drifts play an essential role in material assimilation, radiation dynamics, and plasma evolution. SPI simulations for the AUG tokamak are ongoing and successively improved towards entirely realistic plasma parameters; they qualitatively reproduce experimentally observed double radiation peaks, suggesting that the first peak originates from the injection location and the second peak from the core. Numerical stabilization, axis singularity treatment, and shock-capturing methods are essential ingredients that allow the carrying out of highly nonlinear mitigated disruption studies with the full MHD and reduced-MHD models, which were previously impossible (with Jorek) [7].

7.8 Treatment of grid singularities in the Hermite-Bézier approximations

Participants: A. Bhole, B. Nkonga, H. Guillard, M. Wu, B. Mourain.

JOREK uses high-order isoparametric bi-cubic Hermite-Bézier finite element method (FEM) to numerically approximate fusion plasma models. One distinguishing feature of JOREK's numerical method is the construction of multi-block, flux-aligned grids with curved elements. Such grids may contain geometrically singular points, such as the polar grid center, where FEM is not well defined. These particular points may act as a source of numerical error, polluting the numerical solution. We have already proposed a numerical treatment for the geometric singularity at the polar grid center encountered in the application of the isoparametric bi-cubic Hermite Bézier FEM and implemented the treatment in JOREK. The treatment applies a set of new basis functions at the polar grid center in the numerical algorithm, where the new basis functions are simply the linear transformations of the original basis functions. The proposed treatment enforces the C1 regularity in the physical space, preserves the order of the accuracy of the interpolation, and improves the stability and accuracy of the numerical approximation near the polar grid center [13].

This year's studies go beyond the cases investigated in the past years and suggest a way to enforce regularity when using meshes containing singular points to interpolate smooth functions. The working context also extends the field of study to higher-order approximations by including bi-quintics interpolations. In practice, we use the fact that the meshing vectors differ for each neighbor element of a singular vertex. Therefore, the meshing vectors will now also contain the element's index. Consequently, the degrees of freedom can also differ for each neighbor element. Nevertheless, the physical state and gradient are shared to enforce the C1-regularity of the interpolations. For the C2-regularity, we also share the Hermitian matrix. This formal description, mathematically consistent, when included inside the Jorek platform, will further improve its use in practical and challenging simulations.

7.9 Numerical methods for shear shallow water model

Participants: Boniface Nkonga, Ashish Bhole, Praveen Chandrashekar *(TIFR, Bangalore)*, A. Bhure *(TIFR, Bangalore)*.

A few years ago, we proposed a complete study of the Riemann problem for a genuinely non-conservative hyperbolic system derived from modeling reasonably turbulent thin flows. This model coincides with the ten-moment model derived from gas kinetics in the presence of a gravitational potential. Although the model is fundamentally non-conservative, the generalized jump conditions are compatible with the conservation of total energy. Consequently, the generalized jump conditions consistent with energy conservation are also independent of the path chosen. In this context, we show that the exact solution of the Riemann problem exists and is unique, except in the case of vacuum. This Riemann solver can work well within the numerical framework of Glim and Godunov but at a prohibitive computational cost. We also propose approximate solvers with two, three, or five waves of discontinuities: HLL, HLLC3, and HLLC5. These different solvers are compatible with the conservation of total energy. However, the averaging strategy included in each finite volume approach violates this fundamental conservation law. The main reason for this is that the square of an average quantity is generally not equal to averaging the square of the same amount. Moreover, some fundamental structures of the model also need to be preserved. For example, 1D solutions split into two sub-systems with a one-way coupling between a conservative and a non-conservative sub-systems.

This year's efforts on this subject have focused on strengthening the total energy conservation and the structure-preserving properties at the numerical level to ensure the numerical method converges to the exact solution of the Riemann problem. At least two such strategies have been published in recent years. One explicitly discretizes the conservation equation and uses it to adjust the numerical dissipation of non-conservative variables properly. However, this strategy takes advantage of the direction splitting and does not apply to unstructured meshes. The other approach uses a variable that ensures the symmetry of the stress tensor at the discrete level. It constructs the numerical dissipation coefficient in such a way as to verify the conservation of energy at the discrete level. We are developing a third approach applicable to unstructured meshes that can incorporate multi-wave Riemann solvers. This year, we have some encouraging preliminary results. But we still need to find an optimal solution to this challenging problem.

7.10 Sediment transport in the shear shallow water framework.

Participants: Boniface Nkonga, Ashish Bhole, Praveen Chandrashekar *(TIFR, Bangalore)*, J. Iroume, A. Ngatcha, R. Onguéné, A. Djifendjou.

Several shallow water (SW) based sediment transport models have come into the literature over the last decades. These classical averaged sediment transport models (STM) based on shallow water equations describe the hydromorphodynamic process, assuming that there is no shear velocity along the vertical direction. Mainly, these models do not feel the phase shift between the flow and the bed sediment's movement. The dynamic properties of the coastal flows for tropical rivers of interest are far from the SW assumptions. Our short- and medium-term objective is to revise sediment transport models to include the indispensable shear effects.

The classical models often combined the Exner strategy with the Grass approximations in the framework of the shallow water averaging. This year's work focuses on deriving suitable formulations for Shear Shallow Water (SSW) regimes. The derived model includes the distortion (fluctuation with great correlation lengths) that creates the turbulence., We consider the shift between fluid and sediment velocity (phase-lag) near the bed. The proposed theory significantly reduces the modeling errors observed in several sediment transport models based on nonhomogeneous shallow water equations. It has excellent potential to increase the predictive power of sediment transport models in rivers, lakes, coastal flows, and ocean basins. The proposed theory improves several existing sediment transport theories recently developed in the literature, and we now have some confidence for future applications.

7.11 Parameter identification for a MHD model

Participants: Alexandre Viera, Didier Auroux, Hervé Guillard, Florence Marcotte.

The goal of this study is to identify different parameters used in a reduced MHD model. We mainly focus in two directions. Firstly, the identification of the fluid viscosity and magnetic resistivity. Secondly, the identification of an initial condition leading to a super-critical unstable equilibrium. In order to solve these problems, optimal control techniques will be used. This mainly uses two ingredients: the computation of the gradient of the cost functional, and algorithms to solve optimization problems. In this regard, we used the Tapenade software in order to differentiate a code implementing a reduced MHD model using Hsieh-Clough-Tocher finite elements. This approach lets us compute the exact gradient of a cost functional with respect to the discretization scheme, and not an approximation computed through a discretization of the continuous adjoint equation. The computed gradient obtained is then used in an optimisation loop to minimize a cost functional measuring the distance between the computed and a target solution. Numerical experiments show that the optimization problem converges and allows to recover the value of the viscosity and resistivity corresponding to the target solution.

7.12 Nonlinear acoustic streaming in standing waves

Participants: Hervé Guillard, Argiris Delis *(TUC)*, V. Mandikas *(TUC)*.

Acoustic streaming is a secondary mean steady flow generated by and superimposed on a primary oscillatory flow. When a compressible fluid experiences a high-frequency oscillation (e.g. from a sound source) the nonlinear interactions can often lead to a pattern of time-dependent vortical flows or steady circulations in the flow field. A numerical investigation of the acoustic streaming motion (of the Rayleigh type) in a compressible gas inside two-dimensional rectangular enclosure has been realized last year and published this year [9]

7.13 A robust, discrete-gradient procedure for optimisation with time-dependent PDE and norm constraints

Participants: Didier Auroux, Florence Marcotte.

Joint work with Paul Mannix (formerly postdoc in the CASTOR team, now a postdoc at Imperial College since January 2023) and Calum Skene (postdoc at University of Leeds). Many physical questions in fluid dynamics can be recast in terms of norm-constrained optimization problems; which in-turn, can be further recast as unconstrained problems on spherical manifolds. Due to the nonlinearities of the governing PDEs, and the computational cost of performing optimal control on such systems, improving the numerical convergence of the optimization procedure is crucial. Borrowing tools from the optimization on manifolds community we outline a numerically consistent, discrete formulation of the direct-adjoint looping method accompanied by gradient descent and line-search algorithms with global convergence guarantees. We numerically demonstrate the robustness of this formulation on three example problems of relevance in fluid dynamics and provide an accompanying library SphereManOpt. Paper accepted for publication in *SMAI Journal of Computational Mathematics*.

7.14 Tayler-Spruit dynamos in simulated radiative stellar layers

Participants: Florence Marcotte, Ludovic Petitdemange *((Observatoire de Paris))*, Christophe Gissinger *((Ecole Normale Supérieure))*, Florentin Daniel *((Ecole Normale Supérieure))*.

The Tayler-Spruit dynamo mechanism has been proposed two decades ago as a plausible mechanism to transport angular momentum in radiative stellar layers. Direct numerical simulations are still needed to understand its trigger conditions and the saturation mechanisms. The present study follows up on a previous paper by the same team [11], where we reported the first numerical simulations of a Tayler-Spruit dynamo cycle. Here we extend the explored parameter space to assess in particular the influence of stratification on the dynamo solutions. We also present numerical verification of theoretical assumptions made in (Spruit 2002), which are instrumental in deriving the classical prescription for angular momentum transport implemented in stellar evolution codes. A simplified radiative layer is modeled numerically by considering the dynamics of a stably-stratified, differentially rotating, magnetized fluid in a spherical shell. Our simulations display a diversity of magnetic field topologies and amplitudes depending on the flow parameters, including hemispherical solutions. The Tayler-Spruit dynamos reported here are found to satisfy magnetostrophic equilibrium and achieve efficient turbulent transport of angular momentum, following Spruit's heuristic prediction. Paper accepted for publication in *Astronomy & Astrophysics*.

8 Bilateral contracts and grants with industry

8.1 UKAEA

Participants: Blaise Faugeras, Cédric Boulbe.

A contract has been signed with the English Tokamak UKAEA STEP for a license and a training session on the use of the code NICE.

8.2 France-Relance

Participants: Boniface Nkonga, Jeaniffer Vides.

As part of the Plan France Relance: financing agreement no. ANR-21-PRRD-0005-01 was signed between Agence Nationale de la Recherche (ANR) and Inria 15/06/2021. Then follows an associated contract between Inria, UCA, and CNRS on the one hand and the LEMMA company on the other. (287 K€). The goal was to explore new adaptive sliding methods for rotating machines. At the same time, the Establishments, the Company, and Ms. Jeaniffer VIDES HIGUEROS entered into, on 25/08/2022, an agreement for the provision of Ms. Jeaniffer VIDES HIGUEROS, an employee of the Company, to the CASTOR project team for 24 months

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

AMFoDUC

Title: Advanced Modeling of Flows in Douala City in the strong Urbanization and Climate

Duration: 2021 -> 31/12/2023

Coordinator: Raphael Onguene (ziongra@yahoo.fr)

Partners:

- Université de Douala (Cameroun)

Inria contact: Boniface Nkonga

Summary: The project will bring together students (masters and PhDs) and experts from Cameroun and France to work towards developing simulation tools for the interaction of fluvial flows and coastal flows along the Gulf of Guinea and particularly the city of Douala. The project will require the development of new mathematical tools in the analysis of the Shear Shallow Water (SSW) model, which will also be beneficial to the team towards developing appropriate understanding of new methodologies specific to the context of Guinea Gulf. Currently, there is no other known project in this direction. We aim to further engage neighboring countries along the Gulf of Guinea towards developing a complete simulation tool and further training in geophysical modeling and simulation. In collaboration with our colleague of Bangalore (India/University Côte d'Azur collaboration), we have developed a numerical framework (platform) bringing together the components that are necessary for the deployment of new applications in the context of the SSW modeling. It takes full advantage of massively parallel computing and can serve as a practical engineering tool. These tools are quite flexible, developed in the Finite volume framework and can deal with structured as well as unstructured meshes. On top of this basis, our objective is to develop additional modules that will take into account the specificities of coastal flows in the Gulf of Guinea and contribute to the Douala Sustainable City project. The collected data will help to design the appropriate scaling to be used in the derivation of simplified modeling to be included in the SSW framework. They will also give some guidelines for strategies of numerical validations.

9.2 International research visitors

9.2.1 Visits of international scientists

- Praveen Chandrashekar from TIFR, Bangalore India, visited the team to work the shear shallow water model

- Raphael Onguene from the IUT of Douala, Cameroon, has participated to the Evaluation workshop of the LIRIMA laboratory.

9.3 European initiatives

9.3.1 Horizon Europe

Cédric Boulbe and Blaise Faugeras participate to the Eurofusion Workpackage WPSA

Boniface Nkonga and Hervé Guillard participate to the EuroFusion workpackage 01 (Tokamak exploitation)TSVV

European Starting Grant Project Circe, PI: Florence Marcotte, funded - launching on January 1st, 2024.

9.4 National initiatives

9.4.1 ANR Sistem

Participants: Didier Auroux, Jacques Blum, Cédric Boulbe, Blaise Faugeras, Francesca Rapetti-Gabellini.

Member of the ANR SISTEM , Oct. 2019 - Sept. 2023 coordinated by the M2P2 Institute of Aix-Marseille Univ. "SImulations with high-order schemes of tranSport and TurbulencE in tokaMak" programme Modeles numeriques 2019, Contact : Francesca Rapetti

10 Dissemination

10.1 Promoting scientific activities

Participants: Boniface Nkonga, Francesca Rapetti, Hervé Guillard.

10.1.1 Scientific events: organisation

Member of the organizing committees B. Nkonga is in the organization board of the ECCOMAS conference

Member of the editorial boards F. Rapetti is in the editorial board of the Springer Journal 'Advances in COmputational Mathematics'

10.1.2 Invited talks

- H. Guillard - B. Nkonga, *Godunov-type methods for geometrically constrained hyperbolic systems : MHD and SSW* , Colloquium in memory of Serguei Godunov, Marseille, 30/11-01/12 2023
- H. Guillard, Invited talk at IISER Thiruvananthapuram, Kerala state, India
- H. Guillard, Invited talk at TIFR, Bangalore, India.
- L. Lamerand and G. Gros, Invited session 25, M2P 2023 conf., Taormina (IT)

10.1.3 Scientific expertise

Boniface Nkonga has prepared an expertise for TIFR, Mumbai, India for a high performance computer acquisition project.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Ecole d'ingénieur, D. Auroux, Mathématiques financières, 66h équivalent TD, niveau M1, Université Côte d'Azur
- Ecole d'ingénieur, D. Auroux, Probabilités et statistiques, 24h équivalent TD, niveau L3, Université Côte d'Azur
- Licence : F. Rapetti, Cours Mathématiques 2,60h équivalent TD, L2, Université Côte d'Azur
- Licence : F. Rapetti, TD Mathématiques 2,12h équivalent TD, L2, Université Côte d'Azur
- Licence : F. Rapetti, TD Introduction au calcul scientifique, 24h équivalent TD, L2, Université Côte d'Azur
- Licence : F. Rapetti, TP Introduction au calcul scientifique, 40h équivalent TD, L2, Université Côte d'Azur
- Ecole d'ingénieur, C. Boulbe, Analyse Numérique 1, 45.5h équivalent TD, niveau L3, Université Côte d'Azur
- Ecole d'ingénieur, C. Boulbe, Analyse numérique 2, 45.5h équivalent TD, niveau L3, Université Côte d'Azur
- Ecole d'ingénieur, C. Boulbe, Algèbre linéaire et Matlab, 26h équivalent TD, Université Côte d'Azur
- Ecole d'ingénieur, C. Boulbe et D. Auroux, Projet 1, 48h équivalent TD, Université Côte d'Azur
- Licence : A. Sangam, Fondements Mathématiques 1, 24 h, Semestre 1 de la Licence, Université Côte d'Azur, France
- Licence : A. Sangam, Fondements Mathématiques 2, 40 h, Semestre 2 de la Licence, Université Côte d'Azur, France
- Licence : A. Sangam, Fondements Mathématiques 3, 28 h, Semestre 3 de la Licence, Université Côte d'Azur, France
- Master : A. Sangam, Analyse de Fourier et Distributions, 36 h, Semestre 2 des Masters Mathématiques Fondamentales/Mathématiques Pures et Appliquées, Université Côte d'Azur, France
- Master : A. Sangam, Optimisation et Éléments Finis, 16 h, Semestre 2 des Masters Mathématiques Fondamentales/Mathématiques Pures et Appliquées, et Ingénierie Mathématique, Université Côte d'Azur, France
- Master : A. Sangam, Numerical Approximation of Hyperbolic Systems of Conservation Laws, 40 h, Semestre 3 du Master Mathématiques Pures et Appliquées, Université Côte d'Azur, France
- Ecole d'ingénieur/Master: B. Nkonga, Méthode des éléments finis, 24h, M2, Polytech Nice Sophia, Université Côte d'Azur
- Ecole d'ingénieur/Master: B. Nkonga, Eléments finis mixtes, 24h, M2, Polytech Nice Sophia, Université Côte d'Azur
- Ecole d'ingénieur/Master, H. Guillard, Développement durable et enjeux de gouvernance, 16h, M2, Polytech Nice Sophia, Université Côte d'Azur

10.2.2 Supervision

- Internship of Salma JANATI: Development and implementation of a numerical scheme for evolution PDE of electron temperature in a tokamak plasma

  Blaise FAUGERAS and Afeintou SANGAM have supervised the Master 1 Internship of Salma JANATI from 1st May to 31st July 2023. The internship consisted in designing and implementing of a numerical scheme of electron temperature evolution PDE in a tokamak plasma taking into account geometry coefficients. The obtained results give insight for future schemes aiming to couple plasma profile and electron temperature equations in a tokamak for evolution of the equilibrium at the diffusion time scale as envisioned in NICE code.

11 Scientific production

11.1 Major publications

[1] J. Blum, C. Boulbe and B. Faugeras. 'Reconstruction of the equilibrium of the plasma in a Tokamak and identification of the current density profile in real time'. In: *Journal of Computational Physics* 231 (2012), pp. 960–980. URL: https://hal.archives-ouvertes.fr/hal-00419608.

[2] D. A. Di Pietro, J. Droniou and F. Rapetti. 'Fully discrete polynomial de Rham sequences of arbitrary degree on polygons and polyhedra'. In: *Mathematical Models and Methods in Applied Sciences* (Aug. 2020). DOI: 10.1142/S0218202520500372. URL: https://hal.archives-ouvertes.fr/hal-02356810.

[3] S. Pamela, A. Bhole, G. Huijsmans, B. Nkonga, M. Hoelzl, I. Krebs and E. Strumberger. 'Extended full-MHD simulation of non-linear instabilities in tokamak plasmas'. In: *Physics of Plasmas* 27.10 (Oct. 2020), p. 102510. DOI: 10.1063/5.0018208. URL: https://hal.inria.fr/hal-02974031.

[4] E. Zampa, A. Alonso Rodríguez and F. Rapetti. 'Using the FES framework to derive new physical degrees of freedom for finite element spaces of differential forms'. In: *Advances in Computational Mathematics* 49.2 (6th Mar. 2023), p. 17. DOI: 10.1007/s10444-022-10001-3. URL: https://hal.science/hal-03884149.

11.2 Publications of the year

International journals

[5] A. Alonso Rodríguez, L. Bruni Bruno and F. Rapetti. 'Whitney edge elements and the Runge phenomenon'. In: *Journal of Computational and Applied Mathematics* 427 (Aug. 2023), p. 115117. DOI: 10.1016/j.cam.2023.115117. URL: https://hal.science/hal-03893138.

[6] A. Bhole, H. Guillard, B. Nkonga and F. Rapetti. 'Coupling finite elements of class C1 on composite curved meshes for second order elliptic problems'. In: *International Journal for Numerical Methods in Fluids* (8th Oct. 2023). DOI: 10.1002/fld.5241. URL: https://hal.science/hal-03913143.

[7] A. Bhole, B. Nkonga, J. Costa, G. Huijsmans, S. Pamela and M. Hoelzl. 'Stabilized bi-cubic Hermite Bézier finite element method with application to Gas-plasma interactions occurring during massive material injection in Tokamaks'. In: *Computers & Mathematics with Applications* 142 (July 2023), pp. 225–256. DOI: 10.1016/j.camwa.2023.04.034. URL: https://hal.science/hal-03811224.

[8] C. Boulbe, B. Faugeras, G. Gros and F. Rapetti. 'Tokamak free-boundary plasma equilibrium computations in presence of non-linear materials'. In: *Journal of Scientific Computing* 96 (Aug. 2023). DOI: 10.1007/s10915-023-02265-8. URL: https://hal.science/hal-03423469.

[9] A. Delis, V. Mandikas and H. Guillard. 'Numerical simulation of acoustic streaming in standing waves'. In: *Computers & Mathematics with Applications* 152 (Dec. 2023), pp. 199–220. DOI: 10.1016/j.camwa.2023.10.027. URL: https://inria.hal.science/hal-04281580.

[10] A. R. N. Ngatcha and B. Nkonga. 'Sediment transport models in Generalized shear shallow water flow equations'. In: *Applications in Engineering Science* 15 (Sept. 2023), p. 100148. DOI: 10.1016/j.apples.2023.100148. URL: https://hal.science/hal-04403617.

[11] L. Petitdemange, F. Marcotte and C. Gissinger. 'Spin-down by dynamo action in simulated radiative stellar layers'. In: *Science* 379.6629 (20th Jan. 2023), pp. 300–303. DOI: 10.1126/science.abk2169. URL: https://hal.science/hal-03948010.

[12] E. Zampa, A. Alonso Rodríguez and F. Rapetti. 'Using the FES framework to derive new physical degrees of freedom for finite element spaces of differential forms'. In: *Advances in Computational Mathematics* 49.2 (6th Mar. 2023), p. 17. DOI: 10.1007/s10444-022-10001-3. URL: https://hal.science/hal-03884149.

International peer-reviewed conferences

[13] M. Hoelzl, G. T. A. Huijsmans, F. J. Artola, E. Nardon, M. Becoulet, S. Pamela, B. Nkonga, K. Aleynikova, V. Bandaru, H. Bergström, A. Bhole, T. Bogaarts, D. Bonfiglio, A. Cathey, T. Driessen, S. Futatani, G. Hao, F. Hindenlang, I. Holod, D. Hu, S. Hu, N. Isernia, H. Isliker, S. Kim, M. Kong, S. Korving, L. Kos, I. Krebs, S. Lee, L. Meier, V. Mitterauer, N. Nikulsin, R. Ramasamy, J. Reinking, G. Rubinacci, K. Särkimäki, N. Schwarz, C. Sommariva, R. Sparago, W. Tang, F. Vannini, S. Ventre, F. Villone, L. Wang, H.-H. Wang, F. Wieschollek and J. Zielinski. 'Non-linear MHD Modelling of Transients in Tokamaks: Recent Advances with the Jorek Code'. In: *IAEA Fusion Energy Conference*. FEC 2023 - 29th IAEA Fusion Energy Conference. London, United Kingdom, 16th Oct. 2023. URL: https://hal.science/hal-04403692.

Reports & preprints

[14] A. Alonso Rodríguez, J. Camano, E. de Los santos and F. Rapetti. *Basis for high order divergence-free finite element spaces*. 31st Jan. 2024. URL: https://hal.science/hal-02429500.

[15] A. Alonso Rodríguez, J. Camaño, E. de Los Santos and F. Rapetti. *Weights for moments' geometrical localization: a canonical isomorphism*. 14th Oct. 2023. URL: https://hal.science/hal-04443059.

[16] S. Gavrilyuk, B. Nkonga and K.-M. Shyue. *The conduit equation : hyperbolic approximation and generalized Riemann problem*. 4th Jan. 2024. URL: https://hal.science/hal-04377682.

[17] S. Kerboua-Benlarbi, R. Nouailletas, B. Faugeras, E. Nardon and P. Moreau. *Magnetic control of WEST plasmas through deep reinforcement learning*. Sept. 2023. URL: https://hal.science/hal-04393963.

[18] S. Tiwari, B. Nkonga, P. Chandrashekar and S. Gavrilyuk. *Finite volume approximations of shear shallow water model on unstructured grids*. 18th Jan. 2024. URL: https://hal.science/hal-04403870.

11.3 Cited publications

[19] M. Hoelzl, G. Huijsmans, S. Pamela, M. Becoulet, E. Nardon, F. Artola, B. Nkonga, C. Atanasiu, V. Bandaru, A. Bhole et al. *The JOREK non-linear extended MHD code and applications to large-scale instabilities and their control in magnetically confined fusion plasmas*. 2020. arXiv: 2011.09120 [physics.plasm-ph].