

RESEARCH CENTRE

**Inria Centre  
at the University of Bordeaux**

IN PARTNERSHIP WITH:

**Université de Bordeaux, Institut  
Polytechnique de Bordeaux, Naval Group,  
CNRS**

2023

ACTIVITY REPORT

Project-Team  
ASTRAL

**Advanced Statistical inference And  
control**

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB)

**DOMAIN**

**Applied Mathematics, Computation and  
Simulation**

**THEME**

**Stochastic approaches**

*Inria*

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## Project-Team ASTRAL

*Creation of the Project-Team: 2021 January 01*

### Keywords

#### Computer sciences and digital sciences

- A3.4. – Machine learning and statistics
  - A3.4.1. – Supervised learning
  - A3.4.2. – Unsupervised learning
  - A3.4.3. – Reinforcement learning
  - A3.4.4. – Optimization and learning
  - A3.4.5. – Bayesian methods
  - A3.4.6. – Neural networks
  - A3.4.7. – Kernel methods
  - A3.4.8. – Deep learning
- A6.1.2. – Stochastic Modeling
- A6.1.3. – Discrete Modeling (multi-agent, people centered)
- A6.2.2. – Numerical probability
- A6.2.3. – Probabilistic methods
- A6.2.4. – Statistical methods
- A6.2.6. – Optimization
- A6.3.3. – Data processing
- A6.3.4. – Model reduction
- A6.3.5. – Uncertainty Quantification
- A6.4. – Automatic control
  - A6.4.1. – Deterministic control
  - A6.4.2. – Stochastic control
  - A6.4.3. – Observability and Controlability
  - A6.4.4. – Stability and Stabilization
  - A6.4.5. – Control of distributed parameter systems
  - A6.4.6. – Optimal control
- A8.2.2. – Evolutionary algorithms
- A8.11. – Game Theory
- A9.2. – Machine learning
- A9.3. – Signal analysis
- A9.6. – Decision support
- A9.7. – AI algorithmics

**Other research topics and application domains**

B1.1.2. – Molecular and cellular biology

B1.2.3. – Computational neurosciences

B2.5.1. – Sensorimotor disabilities

B4.2.1. – Fission

# 1 Team members, visitors, external collaborators

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## 2 Overall objectives

### 2.1 Outline of the research project

The highly interconnected contemporary world is faced with an immense range of serious challenges in statistical learning, engineering and information sciences which make the development of statistical and stochastic methods for complex estimation problems and decision making critical. The most significant challenges arise in risk analysis, in environmental and statistical analysis of massive data sets, as well as in defense systems. From both the numerical and the theoretical viewpoints, there is a need for unconventional statistical and stochastic methods that go beyond the current frontier of knowledge.

*Our approach to this interdisciplinary challenge is based on recent developments in statistics and stochastic computational methods. We propose a work programme which will lead to significant breakthroughs in fundamental and applied mathematical research, as well as in advanced engineering and information sciences with industrial applications with a particular focus on defence applications, in collaboration with Naval Group.*

Many real-world systems and processes are dynamic and essentially random. Examples can be found in many areas like communication and information systems, biology, geophysics, finance, economics, production systems, maintenance, logistics and transportation. These systems require dynamic and stochastic mathematical representations with discrete and/or continuous state variables in possibly infinite dimensional space. Their dynamics can be modeled in discrete or continuous time according to different time scales and are governed by different types of processes such as stochastic differential equations, piecewise deterministic processes, jump-diffusion processes, branching and mean field type interacting processes, reinforced processes and self-interacting Markov processes, to name a few. Our interdisciplinary project draws knowledge from information science, signal processing, control theory, statistics and applied probability including numerical and mathematical analysis. The idea is to work across these scientific fields in order to enhance their understandings and to offer an original theory or concept.

Our group mainly focuses on the development of advanced statistical and probabilistic methods for the analysis and the control of complex stochastic systems, as outlined in the following three topics.

- **Statistical and Stochastic modeling:** Design and analysis of realistic and tractable statistical and stochastic models, including measurement models, for complex real-life systems taking into account various random phenomena. Refined qualitative and quantitative mathematical analysis of the stability and the robustness of statistical models and stochastic processes.
- **Estimation/Calibration:** Theoretical methods and advanced computational methodologies to estimate the parameters and the random states of the model given partial and noisy measurements as well as statistical data sets. Refined mathematical analysis of the performance and the convergence of statistical and stochastic learning algorithms.
- **Decision and Control:** Theoretical methods and advanced computational methodologies for solving regulation and stochastic optimal control problems, including optimal stopping problems and partially observed models. Refined mathematical analysis of the long time behavior and the robustness of decision and control systems.

These three items are by no means independent.

- Regarding the interdependence between the modeling aspects and the estimation/calibration/control aspects, it must be emphasized that when optimizing the performance of a partially observed/known stochastic system, the involved mathematical techniques will heavily depend on the underlying mathematical characteristics and complexity of the model of the state process and the model of the observation process. The main difficulty here is to find a balance between complexity and feasibility/solvability. The more sophisticated a model is, the more complicated the statistical inference and optimization problems will be to solve.
- The interdependence that arises between estimation/calibration and the optimal control can be summarised as follows. When the decision-maker has only partial information on the state process,

it is necessary to assume that the admissible control policies will depend on the filtration generated by the observation process. This is a particularly difficult optimisation problem to solve. Roughly speaking, by introducing the conditional distribution of the state process, the problem can then be reformulated in terms of a fully observed control problem. This leads to a separation of estimation and control principle, i.e. the estimation step is carried out first and then the optimisation. The price to be paid for this new formulation is an enlarged state space of infinite dimension. More precisely, in addition to the observable part of the state, a probability distribution enters the new state space which defines the conditional distribution of the unobserved part of the state given the history of the observations.

Solving such global optimization problems remain an open problem and is recognized in the literature as a very difficult challenge to meet.

One of the fundamental challenges we will address is to develop estimation/calibration and optimal control techniques related to general classes of stochastic processes in order to deal with real-world problems. Our research results will combine, mathematical rigour (through the application of advanced tools from probability, statistics, measure theory, functional analysis and optimization) with computational efficiency (providing accurate and applicable numerical methods with a refined analysis of the convergence). Thus, the results that we will obtain in this research programme will be of interest to researchers in the fields of stochastic modeling, statistics and control theory both for the theoretical and the applied communities. Moreover, the topics studied by Naval Group, such as target detection, nonlinear filtering, multi-object tracking, trajectory optimization and navigation systems, provide a diverse range of application domains in which to implement and test the methodologies we wish to develop.

The final goal is to develop a series of reliable and robust softwares dedicated to statistical and stochastic learning, as well as automated decision and optimal control processes. The numerical codes are required to be both accurate and fast since they are often elements of real time estimation and control loops in automation systems. In this regard, the research topics proposed by Naval Group will provide a natural framework for testing the efficiency and robustness of the algorithms developed by the team.

From our point of view, this collaboration between the INRIA project team and Naval Group offers new opportunities and strategies to design advanced cutting-edge estimation and control methodologies.

## 2.2 Approach and methodologies

The types of learning and control methodologies developed by the team differ in their approach as well as in the problems that they are intended to solve. They can be summarised by the following three sets of interdependent methodologies.

- **Statistical learning:** Regression, clustering, volume and dimensionality reduction, classification, data mining, training sets analysis, supervised and unsupervised learning, active and online learning, reinforcement learning, identification, calibration, Bayesian inference, likelihood optimisation, information processing and computational data modeling.
- **Stochastic learning:** Advanced Monte Carlo methods, reinforcement learning, local random searches, stochastic optimisation algorithms, stochastic gradients, genetic programming and evolutionary algorithms, interacting particle and ensemble methodologies, uncertainty propagation, black box inversion tools, uncertainty propagation in numerical codes, rare event and default tree simulation, nonlinear and high dimensional filtering, prediction and smoothing.
- **Decision and control:** Markov decision processes, piecewise deterministic Markov processes, stability, robustness, regulation, optimal stopping, impulse control, stochastic optimal control including partially observed problems, games, linear programming approaches, dynamic programming techniques.

All team members of the project work at the interface of the these three areas. This joint research project between INRIA and Naval Group is a natural and unprecedented opportunity to embrace and push the frontiers of the applied and theoretical sides of these research topics in a common research team.



Despite some recent advances, the design and the mathematical analysis of statistical and stochastic learning tools, as well as automated decision processes, is still a significant challenge. For example, since the mid-1970s nonlinear filtering problems and stochastic optimal control problems with partial observations have been the subject of several mathematical studies, however very few numerical solutions have been proposed in the literature.

Conversely, since the mid-1990s, there has been a virtual explosion in the use of stochastic particle methods as powerful tools in real-world applications of Monte Carlo simulation; to name a few, particle filters, evolutionary and genetic algorithms and ensemble Kalman filters. Most of the applied research in statistics, information theory and engineering sciences seems to be developed in a completely blind way with no apparent connections to the mathematical counterparts.

This lack of communication between the fields often produces a series of heuristic techniques often tested on reduced or toy models. In addition, most of these methodologies do not have a single concrete industrial application nor do they have any connection with physical problems.

As such, there exists a plethora of open mathematical research problems related to the analysis of statistical learning and decision processes. For instance, a variety of theoretical studies on particle algorithms, including particle filters and sequential Monte Carlo models are often based on ad-hoc and practically unrealistic assumptions for the kinds of complex models that are increasingly emerging in applications.

*The aim of this project is to fill these gaps with an ambitious programme at the intersection of probability, statistics, engineering and information sciences.*

One key advantage of the project is its interdisciplinary nature. Combining techniques from pure and applied mathematics, applied probability and statistics, as well as computer science, machine learning, artificial intelligence and advanced engineering sciences enables us to consider these topics holistically, in order to deal with real industrial problems in the context of risk management, data assimilation, tracking applications and automated control systems. The overarching aim of this ambitious programme is to make a breakthrough in both the mathematical analysis and the numerical aspects of statistical learning and stochastic estimation and control.

### 2.3 Innovation and industrial transfer

Fundamentally, our team is not driven by a single application. The reasons are three-fold. Firstly, the robustness and transferability of our approaches means that the same statistical or stochastic learning algorithms can be used in a variety of application areas. On the other hand every application domain offers a series of different perspectives that can be used to improve the design and performances of our techniques and algorithms. Last but not least, industrial applications, including those that arise in defence, require specific attention. As such, we use a broad set of stochastic and statistical algorithms to meet these demands.

This research programme is oriented towards concrete applications with significant potential industrial transfers on three central problems arising in engineering and information and data sciences, namely, risk management and uncertainty propagation, process automation, and data assimilation, tracking and guidance. Our ultimate goal is to bring cutting edge algorithms and advanced statistical tools to industry and defence. The main application domains developed by the team are outlined below:

- **Risk management and uncertainty propagation:** Industrial and environmental risks, fault diagnostics, phase changes, epidemiology, nuclear plants, financial ruin, systemic risk, satellite debris collisions.
- **Process automation:** Production maintenance and manufacturing planning, default detection, integrated dynamics and control of distributed dynamical systems, multi-object coordination, automatic tuning of cochlear implants, classification of EGG signals.
- **Data assimilation, tracking and guidance:** Target detection and classification, nonlinear filtering and multi-object tracking, multiple sensor fusions, motion planning, trajectory optimization, design of navigation systems.

The main objectives and challenges related to the three application domains discussed above will be developed in section 4. The latter application domain will be developed in collaboration with Naval

Group. The reader is referred to section 4.1 for a description of this collaboration and to sections 3.2 and 3.3 for the theoretical aspects that will be carried out by the team in relation to these topics. Specific details on the particular techniques used to tackle the estimation and tracking problems in the context of the collaboration with Naval Group will remain confidential.

### 3 Research program

This section describes the different challenges we intend to address in the theoretical and numerical aspects of statistical/stochastic learning and optimal control. It will be difficult to convey the full complexity of the various topics and to provide a complete overview through a detailed timetable. Nevertheless, we will explain our motivation and why we think it is imperative to address these challenges. We will also highlight the technical issues inherent to these challenges, as well as the difficulties we might expect.

We are confident that the outcomes of this scientific project will lead to significant breakthroughs in statistical/stochastic learning and optimal control with a special emphasis on applications in the defence industry in collaboration with Naval Group. In this respect, we would like to quote Hervé Guillou, CEO of Naval Group, on the occasion of the signing of the partnership agreement between INRIA and Naval Group on December 10, 2019: *"This partnership will enable Naval Group to accelerate its innovation process in the fields of artificial intelligence, intelligence applied to cyber and signal processing. This is a necessity given the French Navy's need for technological superiority in combat and the heightened international competition in the naval defence field..."*

One of our greatest achievements would undoubtedly be to meet these challenges with Naval Group, particularly those related to the fields of statistical/stochastic learning and control. We could not dream of a better outcome for our project.

#### 3.1 Statistical learning

Permanent researchers: M. Chavent, P. Del Moral, F. Dufour, A. Genadot, E. Horton, P. Legrand, J. Saracco.

Regarding statistical learning, some of the objectives of the team is to develop dimension reduction models, data visualization, non-parametric estimation methods, genetic programming and artificial evolution. These models/methodologies provide a way to understand and visualize the structure of complex data sets. Furthermore, they are important tools in several different areas of research, such as data analysis and machine learning, that arise in many applications in biology, genetics, environment and recommendation systems. Of particular interest is the analysis of classification and clustering approaches and semi-parametric modeling that combines the advantages of parametric and non-parametric models, amongst others. One major challenge is to tackle both the complexity and the quantity of data when working on real-world problems that emerge in industry or other scientific fields in academia. Of particular interest is to find ways to handle high-dimensional data with irrelevant and redundant information.

Another challenging task is to take into account successive arrivals of information (data stream) and to dynamically refine the implemented estimation algorithms, whilst finding a balance between the estimation precision and the computational cost. One potential method for this is to project the available information into suitably chosen lower dimensional spaces.

For regression models, sliced inverse regression (SIR) and related approaches have proven to be highly efficient methods for modeling the link between a dependent variable (which can be multidimensional) and multivariate covariates in several frameworks (data stream, big data, etc.). The underlying regression model is semi-parametric (based on a single index or on multiple indices that allow dimension reduction). Currently, these models only deal with quantitative covariates. One of the team's goals is to extend these regression models to mixed data, i.e. models dealing with quantitative and categorical covariates. This generalization would allow one to propose discriminant analysis to deal with mixed data. Extension of sparse principal component analysis (PCA) to mixed data is also another challenge. One idea is to take inspiration from the underlying theory and method of recursive SIR and SIR approaches for data stream in order to adapt them to commonly used statistical methods in multivariate analysis (PCA, discriminant analysis, clustering, etc.). The common aim of all these approaches is to estimate lower dimensional subspaces whilst minimizing the loss of statistical information. Another important aspect of data stream

is the possible evolution in time of the underlying model: we would like to study break(s) detection in semi-parametric regression model, the breakdown being susceptible to appear in the parametric part or in the functional part of the regression model. The question of selecting covariates in regression modelling when we deal with big data is a fundamental and difficult problem. We will address this challenge using genetic programming and artificial evolution. Several directions are possible: for instance, improve, via genetic algorithms, the exploration of the covariate space in closest submodel selection (CSS) method or study optimization problems that simultaneously take into account variable selection, efficiency of estimation and interpretability of the model. Another important question concerns the detection of outliers that will disturb the estimation of the model, and this is not an obvious problem to deal with when working with large, high dimensional data.

In multivariate data analysis, an objective of the team is to work on a new formulation/algorithm for group-sparse block PCA since it is always important to take into account group information when available. The advantage of the group-sparse block PCA is that, via the selection of groups of variables (based on the synthetic variables), interpretability of the results becomes easier. The underlying idea is to address the simultaneous determination of group-sparse loadings by block optimization, and the correlated problem of defining explained variance for a set of non-orthogonal components. The team is also interested in clustering of supervised variables, the idea being to construct clusters made up of variables correlated with each other, which are either well-linked or not-linked to the variable to be explained (which can be quantitative or qualitative).

Another way to study the links between variables is to consider conditional quantiles instead of conditional expectation as is the case in classical regression models. Indeed, it is often of interest to model conditional quantiles, particularly in the case where the conditional mean fails to take into account the impact of the covariates on the dependent variable. Moreover, the quantile regression function provides a much more comprehensive picture of the conditional distribution of a dependent variable than the conditional mean function. The team is interested in the non parametric estimation of conditional quantile estimation. New estimators based on quantization techniques have been introduced and studied in the literature for univariate conditional quantiles and multivariate conditional quantiles. However, there are still many open problems, such as combining information from conditional quantiles of different orders in order to refine the estimation of a conditional quantile of a given order.

Another topic of interest is genetic programming (GP) and Artificial Evolution. GP is an evolutionary computation paradigm for automatic program induction. GP has produced impressive results but there are still some practical limitations, including its high computational cost, overfitting and excessive code growth. Recently, many researchers have proposed fitness-case sampling methods to overcome some of these problems, with mixed results in several limited tests. Novelty Search (NS) is a unique approach towards search and optimization, where an explicit objective function is replaced by a measure of solution novelty. While NS has been mostly used in evolutionary robotics, the team would like to explore its usefulness in classic machine learning problems.

Another important objective of the team is to implement new R (Matlab/Python) packages or to enrich those existing in the literature with the methods we are going to develop in order to make them accessible to the scientific community.

With respect to our statistical learning research program, the objectives of the team can be divided into mid- and long-term works. Mid-term objectives focus on sparsity in SIR (via soft thresholding for instance) and group-sparse block PCA, the underlying idea being to make the selection of variables or blocks of variables in the regression model or in the data. Taking into account multi-block data in regression models via data-driven sparse partial least squares is also at the heart of our concerns. Coupling genetic algorithms and artificial evolution with statistical modeling issues is also planned. The team has several long-term projects associated with the notion of data stream. Many theoretical and practical problems arise from the possible evolution of the information contained in the data: break detection in the underlying model, balance between precision and computational cost. Another scientific challenge is to extend certain approaches such as SIR to the case of mixed data by incorporating the information provided by the qualitative variables in the associated low dimensional subspaces. Moreover, the team has already worked on clustering of variables for mixed data and the clustering of supervised variables is now planned. Finally the idea of combining information from conditional quantiles of different orders in order to refine the estimation of a given order conditional quantile is still relevant today. It should

be noted that other research themes may appear or become a priority depending on the academic or industrial collaborations that may emerge during the next evaluation period.

**Project-team positioning:** Some topics of the INRIA project teams (STATIFY, CELESTE, MODAL, SEQUEL, CLASSIC) are close to the ASTRAL objectives such as non parametric view of high dimensional data, statistical/machine learning, model selection, clustering, sequential learning algorithms, or multivariate data analysis for complex data. While certain ASTRAL objectives are similar to those of these teams, our approaches are significantly different. For example, in multivariate data analysis of complex data including clustering, our team mainly focuses on a geometric approach for mixed data. We also consider the case of successive arrivals of information in SIR both from the theoretical and numerical point of view. Currently there is no direct competition between our team and other INRIA project teams. However, interactions between ASTRAL and other INRIA teams exist. For instance, ASTRAL and STATIFY collaborations are fruitful with common publications, in particular with S. Girard (STATIFY project team). In the field of multivariate data analysis, the team have interesting discussions with Agrocampus Ouest (Rennes, France) and with H.A.L. Kiers (Groningen University) on a mixed data approach for dimension reduction. Conditional and regression quantiles are very active research fields in France (University of Toulouse, Toulouse School of Economics, University of Montpellier) and around the world (ULB, Belgium; University of Illinois Urbana-Champaign, USA; Open University, UK; Brunel University, UK). The ASTRAL team has for the last four-year period collaborated with D. Paindaveine (ULB, Belgium). In the dimension reduction framework, there is a large international community in Europe, America or Asia working on SIR and related methods. However, to our knowledge, the ASTRAL team was the first to introduce importance of variables and recursive methods in SIR, and the first to adapt the SIR approach to data stream.

### 3.2 Stochastic learning

Permanent researchers: M. Chavent, P. Del Moral, F. Dufour, A. Genadot, E. Horton, D. Laneuville, P. Legrand, A. Nègre, J. Saracco, H. Zhang.

Stochastic particle methodologies have become one of the most active intersections between pure and applied probability theory, Bayesian inference, statistical machine learning, information theory, theoretical chemistry, quantum physics, financial mathematics, signal processing, risk analysis, and several other domains in engineering and computer sciences.

Since the mid-1990s, rapid developments in computer science, probability and statistics have led to new generations of interacting particle learning/sampling type algorithms, such as:

*Particle and bootstrap filters, sequential Monte Carlo methods, self-interacting and reinforced learning schemes, sequentially interacting Markov chain Monte Carlo, genetic type search algorithms, island particle models, Gibbs cloning search techniques, interacting simulated annealing algorithms, importance sampling methods, branching and splitting particle algorithms, rare event simulations, quantum and diffusion Monte Carlo models, adaptive population Monte Carlo sampling models, Ensemble Kalman filters and interacting Kalman filters.*

Since computations are nowadays much more affordable, the aforementioned particle methods have become revolutionary for solving complex estimation and optimization problems arising in engineering, risk analysis, Bayesian statistics and information sciences. The books [36], [37], [43],[55] provide a rather complete review on these application domains.

These topics have constituted some of the main research axes of several of the ASTRAL team members since the beginning of the 1990s. To the best of our knowledge, the first rigorous study on particle filters and the convergence of genetic algorithms as the size of the population tends to infinity seems to be the article [45], published in 1996 in the journal Markov Processes and Related Fields. This paper has opened an avenue of research questions in stochastic analysis and particle methods applications. The uniform convergence of particle filters and ensemble Kalman filters with respect to the time horizon was first seen in [40, 39, 42] and in the more recent article [44]. The first use of particle algorithms and Approximated Bayesian Computation type methodologies in nonlinear filtering seems to have started in [41]. Last but not least, the development of sequential Monte Carlo methodology in statistics was introduced in the seminal article [38].

Despite some recent advances, the mathematical foundation and the design and the numerical analysis of stochastic particle methods is still a significant challenge. For instance, particle filter technology is often combined with Metropolis-Hastings type techniques, or with Expectation Maximization type algorithms. The resulting algorithms are intended to solve high dimensional hidden Markov chain problems with fixed parameters. In this context (despite some recent attempts) the refined convergence analysis of the resulting particle algorithms, including exponential concentration estimates, remains to be developed.

Last but not least, the expectations of their performances are constantly rising in a variety of application domains. These particle methodologies are now expected to deal with increasingly sophisticated models in high dimensions, whilst also allowing for the variables to evolve at different scales. *The overarching aim of this aspect of the programme is to make a breakthrough in both the mathematical analysis and the numerical simulation of stochastic and interacting particle algorithms.*

Today, partly because of the emergence of new mean field simulation methodologies and partly because of the importance of new and challenging high-dimensional problems arising in statistical machine learning, engineering sciences and molecular chemistry, we are observing the following trends:

- A need to better calibrate their performance with respect to the size of the systems and other tuning parameters, including cooling decay rates, local random search strategies, interacting and adaptive search criteria, and population size parameters. One of the main and central objectives is to obtain uniform and non asymptotic precision estimates with respect to the time parameter. These types of uniform estimates need to be developed, supporting industrial goals of enhanced design and confidence of algorithms, risk reduction and improved safety.

- A need for new stochastic and adaptive particle methods for solving complex estimation models. Such models arise in a range of application areas including forecasting, data assimilation, financial risk management and analysis of critical events. This subject is also crucial in environmental studies and in the reliability analysis of engineering automated systems. The complexity of realistic stochastic models in advanced risk analysis requires the use of sophisticated and powerful stochastic particle models. These models go far beyond Gaussian models, taking into account abrupt random changes, as well as non nonlinear dynamics in high dimensional state spaces.

- A need to find new mathematical tools to analyze the stability and robustness properties of sophisticated, nonlinear stochastic models involving space-time interaction mechanisms. Most of the theory on the stability of Markov chains is based on the analysis of the regularity properties of linear integral semigroups. To handle these questions, the interface between the theory of nonlinear dynamical systems and the analysis of measure valued processes needs to be further developed.

From a purely probabilistic point of view, the fundamental and the theoretical aspects of our research projects are essentially based on the stochastic analysis of the following three classes of interacting stochastic processes: *Spatial branching processes and mean-field type interacting particle systems, reinforced and self-interacting processes, and finally random tree based search/smoothing learning processes.*

The first class of particle models includes interacting jump-diffusions, discrete generation models, particle ensemble Kalman filters and evolutionary algorithms. This class of models refers to mean field type interaction processes with respect to the occupation measure of the population. For instance genetic-type branching-selection algorithms are built on the following paradigm: when exploring a state space with many particles, we duplicate better fitted individuals while particles with poor fitness die. The selection is made by choosing randomly better fitted individuals in the population. Our final aim is to develop a complete mean-field particle theory combining the stability properties of the limiting processes as the size of the system tends to infinity with the performance analysis of these particle sampling tools.

The second class of particle models refers to mean field type interaction processes with respect to the occupation measure of the past visited sites. This type of reinforcement is observed frequently in nature and society, where "beneficial" interactions with the past history tend to be repeated. Self interaction gives the opportunity to build new stochastic search algorithms with the ability to, in a sense, re-initialize their exploration from the past, re-starting from some better fitted previously visited initial value. In this context, we plan to explore the theoretical foundations and the numerical analysis of continuous time or discrete generation self-organized systems by combining spatial and temporal mean field interaction mechanisms.

The last generation of stochastic random tree models is concerned with biology-inspired algorithms on paths and excursions spaces. These genealogical adaptive search algorithms coincide with genetic

type particle models in excursion spaces. They have been successfully applied in generating the excursion distributions of Markov processes evolving in critical and rare event regimes, as well as in path estimation and related smoothing problems arising in advanced signal processing. The complete mathematical analysis of these random tree models, including their long time behavior, their propagation of chaos properties, as well as their combinatorial structures are far from complete.

Our research agenda on stochastic learning is developed around the applied mathematical axis as well as the numerical perspective, including concrete industrial transfers with a special focus on Naval Group. From the theoretical side, mid-term objectives are centered around non asymptotic performance analysis and long time behavior of Monte Carlo methods and stochastic learning algorithms. We also plan to further develop the links with Bayesian statistical learning methodologies and artificial intelligence techniques, including the analysis of genetic programming discussed in section 3.1. We also have several long term projects. The first one is to develop new particle type methodologies to solve high dimensional data assimilation problems arising in forecasting and fluid mechanics, as well as in statistical machine learning. We also plan to design stochastic filtering-type algorithms to solve partially observed control problems such as those discussed in section 3.3.

**Project-team positioning:** In the last three decades, the use of Feynman-Kac type particle models has been developed in variety of scientific disciplines, including in molecular chemistry, risk analysis, biology, signal processing, Bayesian inference and data assimilation.

The design and the mathematical analysis of Feynman-Kac particle methodologies has been one of the main research topics of P. Del Moral since the late 1990's [45, 41, 40], see also the books [42, 36, 37, 43] and references therein. These mean field particle sampling techniques encapsulate particle filters, sequential Monte Carlo methods, spatial branching and evolutionary algorithms, Fleming-Viot genetic type particles methods arising in the computation of quasi-invariant measures and simulation of non absorbed processes, as well as diffusion Monte Carlo methods arising in numerical physics and molecular chemistry. The term "particle filters" was first coined in the article [45] published in 1996 in reference to branching and mean field interacting particle methods used in fluid mechanics since the beginning of the 1960s. This article presents the first rigorous analysis of these mean field type particle algorithms.

The INRIA project teams applying the particle methodology developed by ASTRAL include the INRIA project team SIMSMART (rare event simulation as well as particle filters) and the INRIA project team Matherials (applications in molecular chemistry). The project team ASTRAL also has several collaborative research projects with these, teams as well as with researchers from international universities working in this subject, including Oxford, Cambridge, New South Wales Sydney, UTS, Bath, Warwick and Singapore Universities.

### 3.3 Decision and stochastic control

Permanent researchers: P. Del Moral, F. Dufour, A. Genadot, E. Horton, D. Laneuville, O. Marceau, A. Nègre, J. Saracco, H. Zhang.

Part of this research project is devoted to the analysis of stochastic decision models. Many real applications in dynamic optimization can be, roughly speaking, described in the following way: a certain system evolves randomly under the control of a sequence of actions with the objective to optimize a performance function. Stochastic decision processes have been introduced in the literature to model such situations and it is undoubtedly their generic capacity to model real life applications that leads to and continues to contribute to their success in many fields such as engineering, medicine and finance.

In this project we will focus on specific families of models that can be identified according to the following elements: the nature of the time variable (discrete or continuous), the type of dynamics (piecewise deterministic trajectories) and the numbers of decision makers. For one player, the system will be called a *stochastic control process* and for the case of several decision-makers, the name (*stochastic game*) will be used. For ease of understanding, we now provide an informal description of the classes of stochastic processes we are interested in, according to the nature of the time variable.

**Discrete-time models.** In this framework, the basic model can be described by a state space where the system evolves, an action space, a stochastic kernel governing the dynamic and, depending on the

state and action variables, a one-step cost (reward) function. The distribution of the controlled stochastic process is defined through the control policy which is then selected in order to optimize the objective function. This is a very general model for dynamic optimization in discrete-time, which also goes by the name of *stochastic dynamic programming*. For references, the interested reader may consult the following books [31, 33, 46, 47, 49, 50, 51, 52, 57, 56, 59] and the references therein (this list of references is, of course, not exhaustive).

**Continuous-time models.** Most of the continuous-time stochastic processes consist of a combination of the following three different ingredients: stochastic jumps, diffusion and deterministic motions. In this project, we will focus on **non-diffusive models**, in other words, stochastic models for which the randomness appears only at fixed or random times, *i.e.* those combining deterministic motions and random jumps. These stochastic processes are the so-called piecewise deterministic Markov processes (PDMPs) [32, 34, 35, 48, 54, 53, 58]. This family of models plays a central role in applied probability because it forms the bulk of models in many research fields such as, *e.g.* operational research, management science and economy and covers an enormous variety of applications.

These models can be framed in several different forms of generality, depending on their mathematical properties such as the type of performance criterion, full or incomplete state information, with or without constraints, adaptative or not, but more importantly, the nature of the boundary of the state space, the type of dynamic between two jumps and on the number of decision-makers. These last three characteristics make the analysis of the controlled process much more involved.

Part of this project will cover both theoretical and numerical aspects of stochastic optimal control. It is clear that stochastic problems and control games have been extensively studied in the literature. Nevertheless, important challenges remain to be addressed. From the theoretical side, there are still many technical issues that are, for the moment, still unanswered or at most have received partial answers. This is precisely what makes them difficult and requires either the creative transposition of pre-existing methodologies or the development of new approaches. It is interesting to note that one of the feature of these theoretical problems is that they are closely related to practical issues. Solving such problems not only gives rise to challenging mathematical questions, but also allow a better insight into the structure and properties of real practical problems. Theory for applications will be for us the thrust that will guide us in this project. From the numerical perspective, solving a stochastic decision model remains a critical issue. Indeed, except for very few specific models, the determination of an optimal policy and the associated value function is an extremely difficult problem to tackle. The development of computational and numerical methods to get quasi-optimal solutions is, therefore, of crucial importance to demonstrate the practical interest of stochastic decision model as a powerful modeling tool. During the International Conference on Dynamic Programming and Its Applications held at the University of British Columbia, Canada in April 1977, Karl Hinderer, a pioneer in the field of stochastic dynamic programming emphasized that "*whether or not our field will have a lasting impact on science beyond academic circles depends heavily on the success of implemented applications*". We believe that this statement is still in force some forty years later.

The objective of this project is to address these important challenges. They are mainly related to models with general state/action spaces and with continuous time variables covering a large field of applications. Here is a list of topics we would like to study: games, constrained control problems, non additive types of criteria, numerical and computational challenges, analysis of partially observed/known stochastic decision processes. This list is not necessarily exhaustive and may of course evolve over time.

Our research agenda on optimal stochastic control is developed around the applied mathematical axis as well as the numerical perspective, including concrete industrial transfers with a special focus on Naval Group. Our mid-term objectives will focus on the following themes described above: properties of control policies in continuous-time control problems, non additive types of criteria, numerical and computational challenges. Our long-term objectives will focus on the analysis of partially observed/known stochastic control problems, constrained control problems and games.

**Project-team positioning:** There exists a large national/international community working on PDMPs and MDPs both on the theoretical, numerical and practical aspects. One may cite A. Almudevar (University of Rochester, USA), E. Altman (INRIA Team NEO, France), K. Avrachenkov (INRIA Team NEO, France), N. Bauerle (Karlsruhe University, Germany), D. Bertsekas (Massachusetts Institute of Technology, USA), O. Costa (Sao Paulo University, Brazil), M. Davis (Imperial College London, England), E. Feinberg (Stony Brook University, USA), D. Goreac (Université Paris-Est Marne-la-Vallée, France), X. Guo (Zhongshan University, China), O. Hernandez-Lerma (National Polytechnic Institute, Mexico), S. Marcus (University of Maryland, USA), T. Prieto-Rumeau (Facultad de Ciencias, UNED, Spain), A. Piunovskiy (University of Liverpool, England), U. Rieder (Universität Ulm, Germany), J. Tsitsiklis (Massachusetts Institute of Technology, USA), B. Van Roy (Stanford University, USA), O. Vega-Amaya (Universidad de Sonora, Mexico), Y. Zhang (University of Liverpool, England) to name just a few. Many of the colleagues cited above are at the head of research groups which have not been described in detail due to space limitation and so, this list is far from being exhaustive.

To some extent, our team is in competition with the colleagues and teams mentioned above. We emphasize that there exists a long standing collaboration between our group and O. Costa (Sao Paulo University, Brazil) since 1998. In the last 10 years, we have established very fruitful collaborations with T. Prieto-Rumeau (Facultad de Ciencias, UNED, Spain) and A. Piunovskiy (University of Liverpool, England).

Inside INRIA, the team NEO and in particular E. Altman and K. Avrachenkov work on discrete-time MDPs but they are mainly focused on the case of countable (finite) state/action spaces MDPs. From this point of view, our results on this theme may appear complementary to theirs.

## 4 Application domains

It is important to point out that (for the time being) only a sub-group of the academic part of the team collaborates with Naval Group. Initially the topics of interest for Naval Group was focused on filtering and control problems. The academic members of this sub-group are P. Del Moral, F. Dufour, A. Genadot, E. Horton and H. Zhang. It is also important to emphasize that Naval Group is undoubtedly our privileged industrial partner. This collaboration is described in section 4.1. For reasons of confidentiality, this section is not very detailed, in particular it does not mention the timetable and does not detail the technical solutions that will be considered. Our aim in the short term is to integrate the remaining academic team members into the group to work on the themes of interest to NG. A seminar was organized for this purpose in August 2020. The academic members of the team who are not involved in collaboration with NG (M. Chavent, P. Legrand and J. Saracco) have their own industrial collaborations that are described in section 4.2.

### 4.1 Naval Group research activities

Permanent researchers: P. Del Moral, F. Dufour, A. Genadot, E. Horton, D. Laneuville, O. Marceau, A. Nègre and H. Zhang.

An important line of research of the team is submarine passive target tracking. This is a very complicated practical problem that combines both filtering and stochastic control topics. In the context of passive underwater acoustic warfare, let us consider a submarine, called the observer, equipped with passive sonars collecting noisy bearing-only measurements of the target(s). The trajectory of the observer has to be controlled in order to satisfy some given mission objectives. These can be, for example, finding the best trajectory to optimize the state estimation (position and velocity) of the targets, maximize the different targets' detection range and/or minimize its own acoustic indiscretion with respect to these targets, and reaching a way-point without being detected. Let us now describe in more detail some of the topics we intend to work on.

In the case of passive tracking problems, one of the main issues is that the observer must manoeuvre in order to generate observability. It turns out that these manoeuvres are actually necessary but not sufficient to guarantee that the problem becomes observable. In fact, a significant body of the literature pertains to attempting to understand whether this type of problem is solvable. Despite this observability analysis, the following practical questions, which we would like to address in this project, remain challenging:



What kind of trajectory should the observer follow to optimize the estimation of the target's motion? What is the accuracy of that estimate? How to deal with a multitarget environment? How to take into account some physical constraints related to the sonar?

Another aspect of target tracking is to take into account both the uncertainties on the target's measurement and also the signal attenuation due to acoustic propagation. To the best of our knowledge, there are few works focusing on the computation of optimal trajectories of underwater vehicles based on signal attenuation. In this context, we would like address the problem of optimizing the trajectory of the observer to maximize the detection of the acoustic signals issued by the targets. Conversely, given that the targets are also equipped with sonars, how can one optimize the trajectory of the observer itself to keep its own acoustic indiscretion as low as possible with respect to those targets.

It must be emphasized that a human operator can find a suitable trajectory for either of these objectives in the context of a single target. However, if both criteria and/or several targets are taken into account simultaneously, it is hardly possible for a human operator to find such trajectories. From an operational point of view, these questions are therefore of great importance.

Such practical problems are strongly connected to the mathematical topics described in sections 3.2 and 3.3. For example it is clearly related to partially observed stochastic control problems. The algorithmic solutions that we will develop in the framework of submarine passive target tracking will be evaluated on the basis of case studies proposed by Naval Group. Our short-term aim is to obtain explicit results and to develop efficient algorithms to solve the various problems described above.

## 4.2 Other collaborations

Permanent researchers: M. Chavent, P. Legrand and J. Saracco.

For several years, the team has also had strong collaborations with INRAE which is the French National Research Institute for Agriculture, Food and Environment. More precisely, consumer satisfaction when eating beef is a complex response based on subjective and emotional assessments. Safety and health are very important in addition to taste and convenience but many other parameters are also extremely important for breeders. Many models were recently developed in order to predict each quality trait and to evaluate the possible trade-off that could be accepted in order to satisfy all the operators of the beef chain at the same time. However, in none of these quality prediction systems are issues of joint management of the different expectations addressed. Thus, it is vital to develop a model that integrates the sensory quality of meat but also its nutritional and environmental quality, which are expectations clearly expressed by consumers. Our team are currently developing statistical models and machine learning tools in order to simultaneously manage and optimize the different sets of expectations. Combining dimension reduction methodologies, nonparametric quantiles estimation and "Pareto front" approaches could provide an interesting way to address this complex problem. These different aspects are currently in progress.

The team is currently initiating scientific collaboration with the Advanced Data Analytics Group of Sartorius Corporate Research which is an international pharmaceutical and laboratory equipment supplier, covering the segments of Bioprocess Solutions and Lab Products & Services. The current work concerns the development of a partial least squares (PLS) inspired method in the context of multiblocks of covariates (corresponding to different technologies and/or different sampling techniques and statistical procedures) and high dimensional datasets (with the sample size  $n$  much smaller than the number of variables in the different blocks). The proposed method allows variable selection in the  $X$  and in the  $Y$  components thanks to interpretable parameters associated with the soft-thresholding of the empirical correlation matrices (between the  $X$ 's blocks and the  $Y$  block) decomposed using singular values decomposition (SVD). In addition, the method is able to handle specific missing values (i.e. "missing samples" in some covariate blocks). The suggested ddsPLS + Koh Lanta methodology is computationally fast. Some technical and/or theoretical work on this methodology must be naturally pursued in order to further refine this approach. Moreover, another aspect of the future research with Sartorius consists of associating the structures of datasets with the real biological dynamics described, until now, by differential equations and for which the most advanced solutions do not merge with both high dimensional multiblock analysis and these differential equations. Combining these two

approaches in a unified framework will certainly have many applications in industry and especially in the biopharmaceutical production.

Within the framework of the GIS ALBATROS, the team has initiated a scientific collaboration with IMS and THALES. The first topic is focused on the measurement of the cognitive load of a pilot through the development of methods for measuring the regularity of biological signals (Hölderian regularity, Detrended Fluctuation Analysis, etc.). The second topic is dedicated to the development of classification techniques of vessels. The different methods we proposed are based on deep learning, evolutionary algorithms and signal processing techniques and are compared to the approaches in the literature.

## 5 Highlights of the year

### 5.1 Habilitation thesis

This year, Alexandre Genadot defended his habilitation thesis [24]

### 5.2 Awards

We have decided to mention in this section the feedback we received from the expert commission at the end of 2023 (originally in french). Indeed, we are very grateful to the experts for giving up their time to perform a fine anlyase of the ASTRAL team. The English translation is given at the end of this section.

Recommandation CE

Les trois experts relèvent l'excellence des travaux théoriques réalisés par l'ensemble des membres de l'équipe (apprentissage statistique et traitement du signal, systèmes de particules en interaction, décision et contrôle), qui les place à la pointe de la R & D sur leurs domaines respectifs. Fort du succès de la collaboration avec Naval Group, un expert suggère que ce type d'interactions soit plus largement répandu afin d'accélérer le transfert de l'état de l'art académique vers les systèmes opérationnels. L'implication d'un ancien doctorant de l'équipe à TryTree de Start-up Studio, est un autre acte de valorisation fort des résultats de l'équipe. Pour répondre à une critique, partagée par 2 des experts, sur sa visibilité internationale, l'équipe produit et intègre à la fiche de synthèse, les statistiques de téléchargement des packages développés et s'engage à renforcer sa participation à l'organisation d'événements scientifiques internationaux. Enfin, sur le volet formation, l'équipe justifie de concentrer ses efforts vers une diffusion académique plutôt qu'à destination des industriels ou du grand public, par manque de temps et de moyens. Les 3 experts partagent la même préoccupation sur la diversité thématique et la façon dont les membres de l'équipe interagissent entre eux. L'équipe voit cette pluralité plutôt comme un atout, un vecteur de fertilisation croisée, et précise le mode de vie de l'équipe qui ménage beaucoup de temps aux échanges. Toutefois, le départ soudain d'Emma Horton (CRCN recrutée en 2020) de l'équipe les a forcés à repenser leur feuille de route pour la période à venir et souligne d'autant plus l'importance du recrutement de jeunes chercheurs à même de pérenniser les compétences scientifiques de l'équipe. La Commission d'Evaluation félicite l'équipe ASTRAL pour son excellente évaluation. Elle remercie l'équipe et Naval Group pour leur forte implication dans cette collaboration fructueuse, établie depuis 2021. De plus, elle se réjouit du succès avéré de cette collaboration. La CE recommande le renouvellement de l'équipe-commune industrielle ASTRAL pour 4 ans.

Avis CEP

ASTRAL, équipe commune avec Naval Group, développe des outils de statistique et probabilité pour l'analyse, l'optimisation et le contrôle de systèmes complexes. L'équipe est structurée sur 3 axes : la modélisation statistique / stochastique; l'estimation et l'apprentissage stochastique; l'optimisation et le contrôle. Les rapporteurs ont souligné les très importantes compétences en mathématique de l'équipes, reconnue au niveau national et international, ainsi que l'impressionnante production en termes de résultats théoriques, publications, production logicielle. L'équipe bénéficie d'une importante visibilité internationale avec des invitations, et participation à des comités éditoriaux de revues majeures. Les rapporteurs ont souligné en particulier la grande visibilité des travaux sur les approches Monte-Carlo particulières, qui font vraiment référence dans le domaine. L'exposé en CEP de François Dufour a rappelé les thèmes de ASTRAL quelques résultats, qui montrent la profondeur du travail, l'interaction entre les axes, et les applications nombreuses avec un riche portefeuille de collaborations industrielles et externes

: Naval Group (évidemment); SARTORIUS; SAFRAN; THALES (avec 3 brevets issus de la collaboration); ONERA; INRAE; INSERM; etc. L'exposé a aussi rappelé l'initiative de startup autour du projet trytree, avec une incubation démarrée en 2022. Les réponses aux remarques des rapporteurs ont éclairé les relations et les interactions entre les différents axes, l'animation interne de l'équipe, l'intérêt de la richesse en thématiques de recherche, et son impact sur la production d'ASTRAL. Des éléments factuels ont souligné l'impact et la visibilité internationale importante de tous les axes dans l'équipe, non seulement via les nombreuses invitations et publications, mais aussi (surtout pour l'axe 1) l'impact des package R open source dont le nombre de téléchargements (par package) se compte par plusieurs dizaines de milliers. Quant aux commentaires sur le cycle de vie et l'âge, l'équipe mentionne être vigilante à son évolution en termes de forces vives et consciente de l'importance et intérêt de nouveaux recrutements. Le CEP a apprécié la très grande actualité des thèmes traités par l'équipe, qui pourrait intéresser beaucoup la communauté IA au sens large, si présentés avec un accent plus fort sur les liens possibles avec cette communauté. En conclusion, le CEP félicite ASTRAL pour son excellente évaluation et soutient très chaleureusement son renouvellement.

English version:

The three experts note the excellence of the theoretical work carried out by all team members (statistical learning and signal processing, interacting particle systems, decision and control), which places them at the cutting edge of R and D in their respective fields. Building on the success of the collaboration with Naval Group, one expert suggests that this type of interaction should be more widespread, to accelerate the transfer of academic state-of-the-art to operational systems. The involvement of one of the team's former PhD students in Start-up Studio's TryTree is another strong act of valorization of the team's results. In response to a criticism, shared by 2 of the experts, of the team's international visibility, the team has included download statistics for the packages it has developed in its summary report, and is committed to stepping up its involvement in the organization of international scientific events. Finally, with regard to training, the team justifies focusing its efforts on academic dissemination rather than on industry or the general public, for lack of time and resources. The 3 experts share the same concern about thematic diversity and the way team members interact with each other. The team sees this plurality more as an asset, a vector for cross-fertilization, and points to the team's way of life, which allows plenty of time for exchanges. However, the sudden departure of Emma Horton (CRCN recruited in 2020) from the team has forced them to rethink their roadmap for the coming period, and underlines all the more the importance of recruiting young researchers capable of perpetuating the team's scientific skills. The Evaluation Committee congratulates the ASTRAL team on its excellent evaluation. It thanks the team and Naval Group for their strong commitment to this fruitful collaboration, established in 2021. It is also delighted with the proven success of this collaboration. The EC recommends the renewal of the ASTRAL joint industrial team for a further 4 years.

ASTRAL, a joint team with Naval Group, develops statistical and probability tools for the analysis, optimization and control of complex systems. The team is structured around 3 axes: statistical/stochastic modeling; estimation and stochastic learning; optimization and control. The reviewers emphasized the team's considerable mathematical skills, which are recognized both nationally and internationally, as well as its impressive output in terms of theoretical results, publications and software production. The team enjoys a high international profile, with invitations to participate in editorial boards of major journals. In particular, the rapporteurs underlined the high visibility of the team's work on particle Monte Carlo approaches, which is a real reference in the field. François Dufour's presentation in CEP recalled some of ASTRAL's themes and results, demonstrating the depth of the work, the interaction between the axes, and the numerous applications with a rich portfolio of industrial and external collaborations: Naval Group (obviously); SARTORIUS; SAFRAN; THALES (with 3 patents resulting from the collaboration); ONERA; INRAE; INSERM; etc. The presentation also recalled the start-up initiative around the trytree project, with incubation starting in 2022. The presentation also recalled the start-up initiative around the trytree project, with incubation starting in 2022. Responses to the rapporteurs' comments shed light on the relationships and interactions between the different axes, the team's internal organization, the value of the richness of the research themes, and their impact on ASTRAL's output. Factual elements underlined the impact and high international visibility of all the team's axes, not only via the numerous invitations and publications, but also (especially for axis 1) the impact of the open source R packages, whose number of downloads (per package) runs into the tens of thousands. As for comments on the team's lifecycle and age, the team mentions that it is vigilant about its evolution in terms of workforce, and is aware of the

importance and interest of new recruits. The CEP appreciated the very topicality of the team's themes, which could be of great interest to the AI community at large, if presented with a stronger emphasis on possible links with this community. In conclusion, the CEP congratulates ASTRAL on its excellent evaluation and warmly supports its renewal.

## 6 New software, platforms, open data

### 6.1 New software

#### 6.1.1 FracLab

**Keyword:** Stochastic process

**Functional Description:** FracLab is a general purpose signal and image processing toolbox based on fractal, multifractal and local regularity methods. FracLab can be approached from two different perspectives: - (multi-) fractal and local regularity analysis: A large number of procedures allow to compute various quantities associated with 1D or 2D signals, such as dimensions, Hölder and 2-microlocal exponents or multifractal spectra.

- Signal/Image processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modeling, segmentation, classification, and synthesis.

**URL:** <https://project.inria.fr/fracLab/>

**Contact:** Jacques Levy-Vehel

**Participants:** Antoine Echelard, Christian Choque-Cortez, Jacques Levy-Vehel, Khalid Daoudi, Olivier Barrière, Paulo Goncalves, Pierrick Legrand

**Partners:** Centrale Paris, Mas

#### 6.1.2 PCAmixdata

**Keyword:** Statistic analysis

**Functional Description:** Mixed data type arise when observations are described by a mixture of numerical and categorical variables. The R package PCAmixdata extends standard multivariate analysis methods to incorporate this type of data. The key techniques included in the package are PCAmix (PCA of a mixture of numerical and categorical variables), PCARot (rotation in PCAmix) and MFAmix (multiple factor analysis with mixed data within a dataset). The MFAmix procedure handles a mixture of numerical and categorical variables within a group - something which was not possible in the standard MFA procedure. We also included techniques to project new observations onto the principal components of the three methods in the new version of the package.

**URL:** <https://cran.r-project.org/web/packages/PCAmixdata/index.html>

**Contact:** Marie Chavent

#### 6.1.3 vimplust

**Keywords:** Clustering, Fair and ethical machine learning

**Functional Description:** vimplust is an R package that implements methods related to sparse clustering and variable importance. The package currently allows to perform sparse k-means clustering with a group penalty, so that it automatically selects groups of numerical features. It also allows to perform sparse clustering and variable selection on mixed data (categorical and numerical features), by preprocessing each categorical feature as a group of numerical features. Several methods for visualizing and exploring the results are also provided.

**URL:** <https://CRAN.R-project.org/package=vimpclust>

**Contact:** Marie Chavent

#### 6.1.4 divdiss

**Name:** divisive monothetic clustering on dissimilarity matrix

**Keywords:** Clustering, Machine learning

**Functional Description:** The `div_diss` function implements a divisive monotopic hierarchical classification algorithm.

**URL:** <https://github.com/chavent/divdiss>

**Contact:** Marie Chavent

## 7 New results

### 7.1 Statistical learning

#### 7.1.1 Optimizing breeding performance through algorithmic approaches to maximize meat quality in livestock

Consumers are now increasingly aware of the impact of meat production on animal welfare and the environment. Simultaneously, there has been a decline in meat consumption and a demand for high-quality meat (in terms of sensory as well as nutritional quality). The study [19] aims to propose a methodological approach that uses breeding practices to estimate meat quality, aiming to achieve optimal quality and meet consumer demand. To achieve this goal, we have developed an updated version of NSGA-II (Non-dominated Sorting Genetic Algorithm II). This algorithm generates a set of candidate solutions, selects the best individuals based on their fitness, and applies genetic operators such as crossover and mutation to generate new offspring. The decision space is defined by the variables  $X$  related to the management of breeding practices, while the objective space  $Y$  represents the variables related to the sensory and/ or nutritional quality of the meat to optimize. To ensure accuracy and precision, the fitness value of each objective is assessed using a multiple linear regression model. An AIC (Akaike Information Criterion) approach is then mobilized to select the most relevant model for each objective. Once a new population is evaluated using the selected models, the Pareto front approach is utilized to identify the non-dominant variables in the multi-objective space. In order to prevent the algorithm from getting trapped in local maximum scenarios, a crowding distance method is employed to maintain population variability and to ultimately reach the global maximum. With this approach, we can generate the best breeding practices for each breed/type of animal and optimize quality. Using the hypervolume approach, we can compare the different optimum front scenarios and recommend, for example, the best breed according to the objectives. In conclusion, this study presents an updated methodological approach for estimating meat quality using breeding practices, which has the potential to improve meat quality and meet consumer demands.

**Participants:** John Albechaalany, Marie-Pierre Ellies-Oury, Jean-François Hocquette, Cécile Berri, Jérôme Saracco (ASTRAL).

#### 7.1.2 What is the impact of the farming system on the quality of the chicken breast meat?

The study presented in [20] aimed to assess the quality characteristics of the four main types of chicken fillets produced in France. The research was conducted on a set of 7,843 fillets collected from commercial slaughterhouses as part of a survey conducted by ITAVI and INRAE. The four main French production systems – Label Rouge, Certified, Standard and Heavy Broiler – were included in the sample, which varied according to animal type, age and/or weight at slaughter, and rearing practices. These systems have

coexisted in France for many years to meet different demands in terms of taste quality and societal expectations, including animal welfare. The study assessed technological quality by ultimate pH, CIELAB colour parameters and curing-cooking yield, sensory quality by juiciness and tenderness after cooking, and nutritional quality by measuring dry matter, protein and fat content. Principal component analysis was performed to identify significant variables that explain chicken fillet quality, and analysis of variance (ANOVA) to investigate relationships between categorical variables. The data also included the time between slaughter and boning of the carcass, which has a significant impact on breast meat tenderness. Label Rouge fillets had the lowest pH and lipid content, but the highest redness and yellowness. The Certified broiler produced the lightest meat. Compared to standard and heavy broilers, Label Rouge and Certified broilers had a higher shear strength, indicating a tougher meat. Heavy broiler breast meat had the highest fat content, but the lowest cooking and processing losses. In addition, the study identified an abattoir effect for each type of broiler, highlighting the significant impact of the slaughter process on breast meat quality. In conclusion, the study highlights the significant impact of the rearing system and genetics on the quality of chicken fillets and the importance of taking into account the characteristics of the animals to optimize the process and ensure optimal meat quality.

**Participants:** John Albechaalany, Saban Yilmaz, Marie-Pierre Ellies-Oury, Marie Bourin, Yann Guyot, Jérôme Saracco (ASTRAL), Jean-François Hocquette, Cécile Berri.

### 7.1.3 HoPSIR: Homogeneous Penalization of Sliced Inverse Regression

In regression, purely parametric approaches require a model that is sometimes complex to set up. Conversely, non-parametric methods suffer when the dimension of the covariate increases as the data points are isolated from each other. Semi-parametric approaches have been proposed to combine the benefits of both approaches. The SIR method is one of them, the parametric part allowing a reduction of dimension. In high dimension, however, SIR is no longer applicable as it requires the inversion of the empirical covariance matrix. Different approaches have been proposed to overcome this technical limitation but none of them has integrated its solution via a statistical model, which is precisely what is proposed in [22]. Through a particular class of functions, the homogeneous functions of positive degree, we introduce a family of prior distributions which allows to build a penalized version of SIR by maximizing the posterior distribution. This approach shows an excellent behaviour on simulations compared to current approaches.

**Participants:** Stéphane Girard, Hadrien Lorenzo.

### 7.1.4 Utility of multivariate data analysis and penalized meta-regression to explore sources of heterogeneity in microbiome meta-analyses

Meta-analysis is a statistical method that quantitatively synthesizes, by calculating a combined result, the results of independent studies addressing a specific research question. The principle is simple: pooling data from several studies increases statistical power. However, a number of conditions must be assessed to ensure that the combined result is not biased and that the conclusions drawn are accurate. A key step is to explore the sources of heterogeneity and look for possible biases. Advances in bioinformatics and next-generation sequencing have led to important advances in the understanding of the role of the microbiota in health. Knowledge development is often based on the conclusions that can be drawn from all published data, i.e. meta-analyses. Yet, in microbiota studies, differences between studies (in terms of pipelines, characteristics, sequencing techniques, sample collection sites, study populations, etc.) can be very high. An exploration of the sources of heterogeneity is essential to determine whether studies, even if they address the same research question, are comparable. In [30], multivariate data analysis methods (such as principal components analysis for quantitative data, multiple correspondence analysis for categorical data, and factor analysis of mixed data, a mixture of the two) as well as penalized

meta-regression (such as the Lasso) are applied to explore heterogeneity in microbiota meta-analyses. Data from recently published microbiome meta-analyses were re-analyzed with the developed tools. In this work, we illustrate the utility of multivariate data analysis methods and penalized meta-regression in exploring sources of heterogeneity in microbiota meta-analyses.

**Participants:** Arthur Saracco, Marie Chavent (ASTRAL), Marta Avalos.

## 7.2 Stochastic learning

### 7.2.1 On the mathematical theory of ensemble (linear-Gaussian) Kalman–Bucy filtering

The purpose of the work presented in [12] and [27] is to present a comprehensive overview of the theory of ensemble Kalman–Bucy filtering for continuous-time, linear-Gaussian signal and observation models. We present a system of equations that describe the flow of individual particles and the flow of the sample covariance and the sample mean in continuous-time ensemble filtering. We consider these equations and their characteristics in a number of popular ensemble Kalman filtering variants. Given these equations, we study their asymptotic convergence to the optimal Bayesian filter. We also study in detail some non-asymptotic time-uniform fluctuation, stability, and contraction results on the sample covariance and sample mean (or sample error track). We focus on testable signal/observation model conditions, and we accommodate fully unstable (latent) signal models. We discuss the relevance and importance of these results in characterising the filter’s behaviour, e.g. its signal tracking performance, and we contrast these results with those in classical studies of stability in Kalman–Bucy filtering. We also provide a novel (and negative) result proving that the bootstrap particle filter cannot track even the most basic unstable latent signal, in contrast with the ensemble Kalman filter (and the optimal filter). We provide intuition for how the main results extend to nonlinear signal models and comment on their consequence on some typical filter behaviours seen in practice, e.g. catastrophic divergence.

**Participants:** Adrian Bishop, Pierre Del Moral (ASTRAL).

### 7.2.2 Robust Kalman and Bayesian Set-Valued Filtering and Model Validation for Linear Stochastic Systems

Consider a linear stochastic filtering problem in which the probability measure specifying all randomness is only partially known. The deviation between the real and assumed probability models is constrained by a divergence bound between the respective probability measures under which the models are defined. This bound defines a so-called uncertainty set. In [13], a recursive set-valued filtering characterization is derived and is guaranteed (with probability one) to contain the true conditional posterior of the unknown, real world, filtering problem when the real world measure is within this uncertainty set. Some filtering approximations and related results are given. The set-valued characterization is related to the problem of robust model validation and model goodness-of-fit statistical hypothesis testing. It is shown how relevant terms involving the innovation sequence (re)appear in multiple settings from set-valued filtering to statistical model evaluation.

**Participants:** Adrian Bishop, Pierre Del Moral (ASTRAL).

### 7.2.3 Self-interacting diffusions: long-time behaviour and exit-problem in the convex case

In [25], we study a class of time-inhomogeneous diffusion: the self-interacting one. We show a convergence result with a rate of convergence that does not depend on the diffusion coefficient. Finally, we establish a so-called Kramers’ type law for the first exit-time of the process from domain of attractions when the landscapes are uniformly convex.

**Participants:** Ashot Aleksian, Pierre del Moral (ASTRAL), Aline Kurtzmann, Julian Tugaut.

#### 7.2.4 A Lyapunov approach to stability of positive semigroups: An overview with illustrations

The stability analysis of possibly time varying positive semigroups on non-necessarily compact state spaces, including Neumann and Dirichlet boundary conditions is a notoriously difficult subject. These crucial questions arise in a variety of areas of applied mathematics, including nonlinear filtering, rare event analysis, branching processes, physics and molecular chemistry. This article [26] presents an overview of some recent Lyapunov-based approaches, focusing principally on practical and powerful tools for designing Lyapunov functions. These techniques include semigroup comparisons as well as conjugacy principles on non-necessarily bounded manifolds with locally Lipschitz boundaries. All the Lyapunov methodologies discussed in the article are illustrated in a variety of situations, ranging from conventional Markov semigroups on general state spaces to more sophisticated conditional stochastic processes possibly restricted to some non-necessarily bounded domains, including locally Lipschitz and smooth hypersurface boundaries, Langevin diffusions as well as coupled harmonic oscillators.

**Participants:** Ashot Aleksian, Marc Arnaudon, Pierre Del Moral (ASTRAL), El Maati Ouhabaz.

### 7.3 Stochastic control and games

#### 7.3.1 Adaptive Discounted Control for Piecewise Deterministic Markov Processes

The main goal in [14] is to study the adaptive infinite-horizon discounted continuous-time optimal control problem of piecewise deterministic Markov processes (PDMPs) with the control acting continuously on the jump intensity  $\lambda$  and on the transition measure  $Q$  of the process. It is assumed that jump parameters ( $\lambda$  and  $Q$ ), as well the continuous and boundary costs ( $C^g$  and  $C^l$  respectively), depend on an unknown parameter  $\beta^*$ . It is shown that the principle of estimation and control holds, that is, the strategy consisting of choosing, at each stage  $n$ , an action according to an optimal stationary policy, where the true but unknown parameter  $\beta^*$  is replaced by its estimated value  $\hat{\beta}_n$ , is asymptotically discount optimal, provided that the sequence of estimators  $\{\hat{\beta}_n\}$  of  $\beta^*$  is strongly consistent, that is,  $\{\hat{\beta}_n\}$  converge to  $\beta^*$  almost surely. In the framework of PDMPs, the so-called discrepancy function depends on the derivative along the flow of the value function as well as on some boundary conditions, which brings new challenges in the analysis of this problem.

**Participants:** O.L.V. Costa, François Dufour (ASTRAL).

#### 7.3.2 Absorbing Markov Decision Processes

In [15], we study discrete-time absorbing Markov Decision Processes (MDP) with measurable state space and Borel action space with a given initial distribution. For such models, solutions to the characteristic equation that are not occupation measures may exist. Several necessary and sufficient conditions are provided to guarantee that any solution to the characteristic equation is an occupation measure. Under the so-called continuity-compactness conditions, we first show that a measure is precisely an occupation measure if and only if it satisfies the characteristic equation and an additional absolute continuity condition. Secondly, it is shown that the set of occupation measures is compact in the weak-strong topology if and only if the model is uniformly absorbing. Several examples are provided to illustrate our results.



**Participants:** François Dufour (ASTRAL), Tomás Prieto-Rumeau.

### 7.3.3 Nash equilibria for total expected reward absorbing Markov games: the constrained and unconstrained cases

In [16], we consider a nonzero-sum  $N$ -player Markov game on an abstract measurable state space with compact metric action spaces. The payoff functions are bounded Caratheodory functions and the transitions of the system are assumed to have a density function satisfying some continuity conditions. The optimality criterion of the players is given by a total expected payoff on an infinite discrete-time horizon. Under the condition that the game model is absorbing, we establish the existence of Markov strategies that are a noncooperative equilibrium in the family of all history-dependent strategies of the players for both the constrained and the unconstrained problems. We obtain, as a particular case of results, the existence of Nash equilibria for discounted constrained and unconstrained game models.

**Participants:** François Dufour (ASTRAL), Tomás Prieto-Rumeau.

### 7.3.4 Minimum Contrast Estimators for Piecewise Deterministic Markov Processes

The main goal of [28] is to study the minimum contrast estimator (MCE) approach for the parameter estimation problem of piecewise deterministic Markov processes (PDMPs), associated to adaptive control problems. It is assumed that the control acts continuously on the jump intensity  $\lambda$  and on the transition measure  $Q$  of the process, as well as on the costs, and that these parameters depend on an unknown parameter  $\beta^*$ . One of our objective is to introduce a minimum contrast estimator  $(\beta_n)_{n \in \mathbb{N}}$  for the family of PDMPs. Sufficient conditions are then presented to ensure that  $(\beta_n)$  is a strongly consistent estimator of  $\beta^*$ . It should be noticed that PDMPs are characterized by a deterministic motion punctuated by random jumps (either spontaneous or due to the flow touching a boundary), which brings new challenges in the analysis of the problem. The paper is concluded with a numerical example for the adaptive discounted control of PDMPs.

**Participants:** Alexandre Genadot (ASTRAL), François Dufour (ASTRAL), Oswaldo Luiz Do Valle Costa.

## 7.4 Signal processing, artificial evolution and neural networks

### 7.4.1 Combining the global trends of DFA or CDFA of different orders

When dealing with the detrended fluctuation analysis (DFA) and its variants such as the higher-order DFA and the continuous DFA (CDFA), removing the mean, integrating the signal and detrending it amount to applying an equivalent linear filter characterized by its frequency response when processing a wide-sense stationary random process. By studying the frequency response of each variant, one can better understand its behavior. Thus, in [17], we first compare the frequency responses of two DFAs whose orders are different. Then, by deriving the higher-order CDFA using Lagrange multipliers, we can show how the frequency responses of the equivalent filters of the CDFA and the DFA are related to each other. Finally, we propose to combine the global trends obtained with the DFA or the CDFA of different orders to derive a new variant. Once again, the behavior is studied using the frequency response of the equivalent filter. Illustrations and comments on ARFIMA processes and Weierstrass functions are also given to evaluate their long range dependence.

**Participants:** Eric Grivel, Jeremie Soetens, Gaetan Colin, Pierrick Legrand (ASTRAL).

### 7.4.2 Evaluation of Real-Time Aliasing Reduction Methods in Neural Networks for Nonlinear Audio Effects Modelling

Neural networks have seen increased popularity in recent years for nonlinear audio effects modelling. Such a task requires sampling and creates high frequency harmonics that can quickly surpass the Nyquist rate, creating aliasing in the baseband. In [18], we study the impact of processing audio with neural networks and the potential aliasing these highly nonlinear algorithms can incur or aggravate. Namely, we evaluate the performance of a number of anti-aliasing methods for use in real-time. Notably, one method of anti-aliasing capable of real-time performance was identified: forced sparsity through network pruning.

**Participants:** Tara Vanhatalo (ASTRAL), Pierrick Legrand (ASTRAL), Myriam Desainte-Catherine, Pierre Hanna, Guillaume Pille.

### 7.4.3 Null or linear-phase filters for the derivation of a new variant of the MSE

The multiscale entropy (MSE) is used in a wide range of applications, especially by physiologists with bio-signals for classification. It consists in estimating the sample entropies of the signal under study and its coarse-grained (CG) versions. The CG process amounts to filtering the signal with an average filter whose order is the scale and then to decimating the filter output by a factor also equal to the scale. In [21], the novelty stands in the way to get the sequences at different scales, by avoiding distortions that can appear during the decimation step. If a low-pass filter is implemented, the cut-off frequency of which is well suited to the decimation factor, the phase distortions induced by the filter must be attenuated as much as possible. Two ways are considered in this paper: 1/ design a linear-phase finite-impulse-response filter with the window method or the Remez algorithm. 2/ design a zero-phase filter from any infinite impulse response filter. Simulations with white noises and  $1/f$  processes are given. The way the sample entropy evolves with the scale is presented. Depending on the filter parameters, one can see how it differs from the evolution obtained with the CG.

**Participants:** Eric Grivel, Bastien Berthelot, Pierrick Legrand (ASTRAL), Gaetan Colin.

## 8 Bilateral contracts and grants with industry

### 8.1 Bilateral contracts with industry

#### Naval Group

**Participants:** Pierre Del Moral, François Dufour, Alexandre Genadot, Emma Horton, Dann Laneuville, Olivier Marceau, Adrien Nègre, Raymond Zhang.

In the application domain, an important research focus of the team is the tracking of passive underwater targets in the context of passive underwater acoustic warfare. This is a very complicated practical problem that combines both filtering and stochastic control issues. This research topic is addressed in collaboration with Naval Group. We refer the reader to the section 4.1 for a more detailed description of this theme.

#### Thales AVS

**Participants:** Bastien Berthelot, Pierrick Legrand.

The collaboration is centered around some contributions to the estimation of the Hurst coefficient and his application on biosignals in the domain of crew monitoring.

### Case Law Analytics

**Participants:** Pierrick Legrand.

Pierrick Legrand is a consultant for the startup Case Law Analytics. The object of the consulting is confidential.

## 8.2 Bilateral Grants with Industry

### Orosys

**Participants:** Tara Vanhatalo, Pierrick Legrand.

Within the framework of Tara Vanhatalo's Cifre PhD thesis on the stochastic modeling of guitar amplifiers, a strong collaboration was established between the company Orosys and the ASTRAL team.

### Arturia

**Participants:** Yann Bourdin, Pierrick Legrand.

Within the framework of Yann Bourdin's Cifre PhD thesis on the stochastic modeling of audio compressor and nonlinear audio effects, a strong collaboration was established between the company Arturia and the ASTRAL team.

## 9 Partnerships and cooperations

**Participants:** Pierre del Moral, François Dufour.

### 9.1 International research visitors

#### 9.1.1 Visits of international scientists

##### Other international visits to the team

**Ajay Jasra**

**Status** researcher

**Institution of origin:** Kaust

**Country:** Arabie Saoudite

**Dates:**

**Context of the visit:** Work with P. del Moral

**Mobility program/type of mobility:** research stay

**George Yin**

**Status** researcher

**Institution of origin:** University of Connecticut

**Country:** USA

**Dates:** November 5-10

**Context of the visit:** Work with F. Dufour

**Mobility program/type of mobility:** research stay

**Oswaldo Luis do Valle Costa**

**Status** researcher

**Institution of origin:** Escola Politecnica, Universidade de Sao Paulo

**Country:** Brasil

**Dates:** September 5-9

**Context of the visit:** Work with F. Dufour

**Mobility program/type of mobility:** research stay

## 9.2 European initiatives

### 9.2.1 Other european programs/initiatives

Project PID2021-122442NB-I00

F. Dufour. "Analysis and control of deterministic, stochastic, and game theoretical dynamical systems" from 01/09/2022 to 31/08/2025. Ministry of Science and Innovation, Spain.

## 9.3 National initiatives

**Naval Group** Astral is a joint INRIA team project with Naval Group. The topic of this collaboration is described in section [4.1](#).

**QuAMProcs of the program *Project Blanc* of the ANR** The mathematical analysis of metastable processes started 75 years ago with the seminal works of Kramers on Fokker-Planck equation. Although the original motivation of Kramers was to « elucidate some points in the theory of the velocity of chemical reactions », it turns out that Kramers' law is observed to hold in many scientific fields: molecular biology (molecular dynamics), economics (modelization of financial bubbles), climate modeling, etc. Moreover, several widely used efficient numerical methods are justified by the mathematical description of this phenomenon.

Recently, the theory has witnessed some spectacular progress thanks to the insight of new tools coming from Spectral and Partial Differential Equations theory.

Semiclassical methods together with spectral analysis of Witten Laplacian gave very precise results on reversible processes. From a theoretical point of view, the semiclassical approach allowed to prove a complete asymptotic expansion of the small eigen values of Witten Laplacian in various situations (global problems, boundary problems, degenerate diffusions, etc.). The interest in the analysis of boundary problems was rejuvenated by recent works establishing links between the Dirichlet problem on a bounded domain and the analysis of exit event of the domain. These results open numerous perspectives of applications. Recent progress also occurred on the analysis of irreversible processes (e.g. on overdamped Langevin equation in irreversible context or full (inertial) Langevin equation).

The above progresses pave the way for several research tracks motivating our project: overdamped Langevin equations in degenerate situations, general boundary problems in reversible and irreversible case, non-local problems, etc.

**Mission pour les initiatives transverses et interdisciplinaires, Défi Modélisation du Vivant, projet MISGIVING** The aim of MISGIVING (Mathematical Secrets penGuins dIVING) is to use mathematical models to understand the complexity of the multiscale decision process conditioning not only the optimal duration of a dive but also the diving behaviour of a penguin inside a bout. A bout is a sequence of successive dives where the penguin is chasing prey. The interplay between the chasing period (dives) and the resting period due to the physiological cost of a dive (the time spent at the surface) requires some kind of optimization.

## 10 Dissemination

### 10.1 Promoting scientific activities

**Participants:** All team members.

#### 10.1.1 Scientific events: organisation

**Member of the organizing committees** Pierrick Legrand started the organization of EA 2024 in Bordeaux in 2023.

**General chair, scientific chair** Pierrick Legrand started the organization of EA 2024 in Bordeaux in 2023 as general chair.

#### 10.1.2 Scientific events: selection

All team members are regular reviewers and PC members for leading conferences in probability and applied statistics, control theory, signal processing and artificial evolution.

#### 10.1.3 Journal

**Member of the editorial boards** P. Del Moral is an Associated Editor in the Annals of Applied Probability, since 2019.

P. Del Moral is an Associated Editor in Foundations of Data Science, since 2018.

P. Del Moral is an Associated Editor in Stochastic Analysis and Applications, since 2001.

F. Dufour was an associate editor for the SIAM Journal of Control and Optimization from 2009 to 2018.

F. Dufour is a corresponding editor for the SIAM Journal of Control and Optimization since 2018.

F. Dufour is an associate editor for the Journal Applied Mathematics and Optimization since 2018.

F. Dufour is an associate editor for the Journal Stochastics since 2018.

F. Dufour was the representative of the SIAM activity group in control and system theory for the journal SIAM News from 2014 to 2020.

F. Dufour was the chair of the selection committee for the 2021 SIAG/CST Best SICON Paper Prize.

F. Dufour is a member of the IFAC Technical Committee TC 1.4 Stochastic Systems.

J. Saracco is member of the Editorial Board of Astrostatistics (specialty section of Frontiers in Astronomy and Space Sciences).

M. Chavent is a member of the editorial board of the collection Pratique R, EDP Sciences.

Pierrick Legrand is the main editor for the Springer LNCS volumes Artificial Evolution since 2009. In 2023, he published the best papers of the conference EA 2022 in [23].

**Reviewer - reviewing activities** All team members are regular reviewers for leading journals in probability and applied statistics, control theory, signal processing and artificial evolution.

#### 10.1.4 Leadership within the scientific community

Pierrick Legrand is the president of the association Evolution Artificielle.

#### 10.1.5 Scientific expertise

J. Saracco is regularly an expert for the HCERES (expertise of French mathematics laboratories) and for the ANRT (expertise of CIFRE thesis applications).

P. Legrand is regularly an expert for the ANR program.

#### 10.1.6 Research administration

**National responsibilities** J. Saracco is treasurer of the French Statistical Society (SFdS) and an elected member of its board.

J. Saracco is an elected member of the national council of universities in applied mathematics (CNU 26).

**Universities and schools** E. Dufour is an invited member of the scientific council of the Institute of Mathematics of Bordeaux.

E. Dufour is member of the council of ENSEIRB MATMECA - Bordeaux INP since 2022.

J. Saracco is an elected member of the Bordeaux INP Board of Studies.

J. Saracco is an elected member of the Board of ENSC Bordeaux INP.

J. Saracco was head of the research team "OptimAl" (mathematical optimisation, random and statistical models) of the Mathematical Institute of Bordeaux (UMR 5251 CNRS).

M. Chavent is a member of the council of the department Sciences de l'ingénierie et du numérique of Bordeaux University.

M. Chavent is an elected member of the scientific council of the Institute of Mathematics of Bordeaux (since 2022).

A. Génadot has been an elected member of the scientific council of the Institute of Mathematics of Bordeaux (2019-2022).

P. Legrand is an elected member of the scientific council of the Institute of Mathematics of Bordeaux (2022-).

P. Legrand was the president of the consultative commission in section CNU 26 of the Institute of Mathematics of Bordeaux (2019-2023).

**Inria** M. Chavent is a member of the CDT (Commission for Technological Development) at Inria Bordeaux since september 2022.

M. Chavent was a member of the Inria Evaluation Committee (2015-2019).

P. Del Moral is a member of the "Bureau du Comité des Projets" of the INRIA Bordeaux-Sud Ouest Research Center, since 2018.

A. Génadot is a member of the CER of Inria Bordeaux.

P. Legrand is a member of the CUMI commission.

## 10.2 Teaching - Supervision - Juries

### 10.2.1 Teaching

- J. Saracco is the head of the engineering department of ENSC, Graduate School of Cognitics (applied cognitive science and technology) which is a Bordeaux INP engineering school.
- Marie Chavent is in charge of the first year of the MIASHS degree program at Université de Bordeaux.
- Alexandre Genadot is in charge of the first year of the MIASHS degree program at Université de Bordeaux.

- Pierrick Legrand is in charge of the mathematics program for the MIASHS degree at Université de Bordeaux and also the head of the MIASHS Licence of the Université de Bordeaux.
- Licence : P. Legrand, Algèbre, 129h, L1, Université de Bordeaux, France.
- Licence : P. Legrand, Espaces Euclidiens, 46,5h, L2, Université de Bordeaux, France.
- DU : P. Legrand, Evolution Artificielle, Big data, 8h, DU, Bordeaux INP, France.
- Engineer School: Signal processing, 54 hours, ENSC, Bordeaux, France.
- Master: Scientific courses, 10 hours, Université de Bordeaux, France.
- Licence : A. Genadot, Bases en Probabilités, 18h, L1, Université de Bordeaux, France.
- Licence : A. Genadot, Projet Professionnel de l'étudiant, 8h, L1, Université de Bordeaux, France.
- Licence : A. Genadot, Probabilité, 30h, L2, Université de Bordeaux, France.
- Licence : A. Genadot, Techniques d'Enquêtes, 10h, L2, Université de Bordeaux, France.
- Licence : A. Genadot, Modélisation Statistiques, 16.5h, L3, Université de Bordeaux, France.
- Licence : A. Genadot, Préparation Stage, 15h, L3, Université de Bordeaux, France.
- Licence : A. Genadot, TER, 5h, L3, Université de Bordeaux, France.
- Licence : A. Genadot, Processus, 16.5h, L3, Université de Bordeaux, France.
- Licence : A. Genadot, Statistiques, 20h, L3, Bordeaux INP, France.
- Master : A. Genadot, Savoirs Mathématiques, 81h, M1, Université de Bordeaux et ESPE, France.
- Master : A. Genadot, Martingales, 29h, M1, Université de Bordeaux, France.
- Licence : F. Dufour, Probabilités et statistiques, 70h, first year of école ENSEIRB-MATMECA, Institut Polytechnique de Bordeaux, France.
- Master : F. Dufour, Approche probabiliste et méthode de Monte Carlo, 24h, third year of école ENSEIRB-MATMECA, Institut Polytechnique de Bordeaux, France.
- Licence : J. Saracco, Probabilités et Statistique, 27h, first year of Graduate Schools of Engineering ENSC-Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- Licence : J. Saracco, Statistique inférentielle et Analyse des données, 45h, first year of Graduate Schools of Engineering ENSC-Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- Licence : J. Saracco, Statistique pour l'ingénieur, 16h, first year of Graduate Schools of Engineering ENSPIMA-Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- Master : J. Saracco, Modélisation statistique, 81h, second year of Graduate Schools of Engineering ENSC-Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- DU : J. Saracco, Statistique et Big data, 45h, DU BDSI (Big data et statistique pour l'ingénieur), Bordeaux INP, France.
- Licence : M. Chavent, Statistique Inférentielle, 18h, L2, Université de Bordeaux, France
- Licence : M. Chavent, Techniques d'Enquêtes, 10h, L2, Université de Bordeaux, France
- Master : M. Chavent, DataMining, 43h, M2, Université de Bordeaux
- Master : M. Chavent, Machine Learning, 58h, Université de Bordeaux,
- DU: M. Chavent, Apprentissage, 12h, DU BDSI, Bordeaux INP, France

## 10.2.2 Juries

All team members are regular member of PhD's and Master's juries.

## 10.3 Popularization

### 10.3.1 Intervention

Pierrick Legrand gave a talk at a "Unité ou café" session.

## 11 Scientific production

### 11.1 Major publications

- [1] P. Del Moral and E. Horton. *A theoretical analysis of one-dimensional discrete generation ensemble Kalman particle filters*. 3rd July 2021. URL: <https://hal.inria.fr/hal-03277374>.
- [2] F. Dufour and T. Prieto-Rumeau. *Stationary Markov Nash equilibria for nonzero-sum constrained ARAT Markov games*. 4th Jan. 2022. URL: <https://hal.inria.fr/hal-03510818>.
- [3] M.-P. Ellies-Oury, D. Durand, A. Listrat, M. Chavent, J. Saracco and D. Gruffat. 'Certain relationships between Animal Performance, Sensory Quality and Nutritional Quality can be generalized between various experiments on animal of similar types'. In: *Livestock Science* 250 (Aug. 2021), p. 104554. DOI: [10.1016/j.livsci.2021.104554](https://doi.org/10.1016/j.livsci.2021.104554). URL: <https://hal.archives-ouvertes.fr/hal-03272625>.
- [4] A. Genadot. 'Contributions à l'étude des processus markoviens déterministes par morceaux et de décision ainsi qu'à l'étude de l'enquête Bourciez'. Université de Bordeaux, 25th Oct. 2023. URL: <https://hal.science/tel-04326411>.
- [5] A. Genadot. 'Quina metodologia per despolhar l'enquèsta Bourciez?' In: *Ièr, deman : diga-m'o dins la lenga*. Montpellier, France, 26th Nov. 2021. URL: <https://hal.inria.fr/hal-03471470>.
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