IN PARTNERSHIP WITH:
CNRS

Université de Strasbourg

Activity Report 2019

Project-Team TONUS

TOkamaks and NUmerical Simulations

IN COLLABORATION WITH: Institut de recherche mathématique avancée (IRMA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Earth, Environmental and Energy Sciences

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Project-Team TONUS

Creation of the Team: 2012 January 01, updated into Project-Team: 2019 May 01

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2. Overall Objectives

2.1. Overall Objectives

TONUS started in January 2014. It is a team of the Inria Nancy-Grand Est center. It is located in the mathematics institute (IRMA) of the University of Strasbourg.

The International Thermonuclear Experimental Reactor (ITER) is a large-scale scientific experiment that aims to demonstrate that it is possible to produce energy from fusion, by confining a very hot hydrogen plasma inside a toroidal chamber, called tokamak. In addition to physics and technology research, tokamak design also requires mathematical modelling and numerical simulations on supercomputers.

The objective of the TONUS project is to deal with such mathematical and computing issues. We are mainly interested in kinetic, gyrokinetic and fluid simulations of tokamak plasmas. In the TONUS project-team we are working on the development of new numerical methods devoted to such simulations. We investigate several classical plasma models, study new reduced models and new numerical schemes adapted to these models. We implement our methods in two software projects: Selalib [1] and SCHNAPS [2] adapted to new computer architectures.

We have strong relations with the CEA-IRFM team and participate in the development of their gyrokinetic simulation software GYSELA. We are involved in two Inria Project Labs, respectively devoted to tokamak mathematical modelling and high performance computing. The numerical tools developed from plasma physics can also be applied in other contexts. For instance, we collaborate with a small company in Strasbourg specialized in numerical software for applied electromagnetism. We also study kinetic acoustic models with the CEREMA and multiphase flows with EDF.

Finally, our topics of interest are at the interaction between mathematics, computer science, High Performance Computing, physics and practical applications.

3. Research Program

3.1. Kinetic models for plasmas

The fundamental model for plasma physics is the coupled Vlasov-Maxwell kinetic model: the Vlasov equation describes the distribution function of particles (ions and electrons), while the Maxwell equations describe the electromagnetic field. In some applications, it may be necessary to take relativistic particles into account, which leads to consider the relativistic Vlasov equation, even if in general, tokamak plasmas are supposed to be non-relativistic. The distribution function of particles depends on seven variables (three for space, three for the velocity and one for time), which yields a huge amount of computation. To these equations we must add several types of source terms and boundary conditions for representing the walls of the tokamak, the applied electromagnetic field that confines the plasma, fuel injection, collision effects, etc.

Tokamak plasmas possess particular features, which require developing specialized theoretical and numerical tools.

Because the magnetic field is strong, the particle trajectories have a very fast rotation around the magnetic field lines. A full resolution would require a prohibitive amount of computation. It is necessary to develop reduced models for large magnetic fields in order to obtain tractable calculations. The resulting model is called a gyrokinetic model. It allows us to reduce the dimensionality of the problem. Such models are implemented in GYSELA and Selalib.

On the boundary of the plasma, the collisions can no more be neglected. Fluid models, such as MagnetoHydroDynamics (MHD) become again relevant. For the good operation of the tokamak, it is necessary to control MHD instabilities that arise at the plasma boundary. Computing these instabilities requires special implicit numerical discretizations with excellent long time behavior.

In addition to theoretical modelling tools, it is necessary to develop numerical schemes adapted to kinetic, gyrokinetic and fluid models. Three kinds of methods are studied in TONUS: Particle-In-Cell (PIC) methods, semi-Lagrangian and fully Eulerian approaches.

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[1] http://selalib.gforge.inria.fr/

[2] http://schnaps.gforge.inria.fr

3.1.1. Gyrokinetic models: theory and approximation

In most phenomena where oscillations are present, we can establish a three-model hierarchy: (i) the model parameterized by the oscillation period, (ii) the limit model and (iii) the two-scale model, possibly with its corrector. In a context where one wishes to simulate such a phenomenon where the oscillation period is small and the oscillation amplitude is not small, it is important to have numerical methods based on an approximation of the two-scale model. If the oscillation period varies significantly over the domain of simulation, it is important to have numerical methods that approximate properly and effectively the model parameterized by the oscillation period and the two-scale model. Implementing two-scale numerical methods (for instance by Frénod et al. [27]) is based on a numerical approximation of the Two-Scale model. These are called of order 0. A Two-Scale Numerical Method is called of order 1 if it incorporates information from the corrector and from the equation of which this corrector is a solution. If the oscillation period varies between very small values and values of order 1, it is necessary to have new types of numerical schemes (Two-Scale Asymptotic Preserving Schemes of order 1 or TSAPS) that preserve the asymptotics between the model parameterized by the oscillation period and the Two-Scale model with its corrector. A first work in this direction has been initiated by Crouseilles et al. [26].

3.1.2. Semi-Lagrangian schemes

The Strasbourg team has a long and recognized experience in numerical methods for Vlasov-type equations. We are specialized in both particle and phase space solvers for the Vlasov equation: Particle-in-Cell (PIC) methods and semi-Lagrangian methods. We also have a long-standing collaboration with CEA Cadarache for the development of the GYSELA software for gyrokinetic tokamak plasmas.

The Vlasov and the gyrokinetic models are partial differential equations that express the transport of the distribution function in the phase space. In the original Vlasov case, the phase space is the six-dimension position-velocity space. For the gyrokinetic model, the phase space is five-dimensional because we consider only the parallel velocity in the direction of the magnetic field and the gyrokinetic angular velocity instead of three velocity components.

A few years ago, Eric Sonnendrücker and his collaborators introduced a new family of methods for solving transport equations in the phase space. This family of methods are the semi-Lagrangian methods. The principle of these methods is to solve the equation on a grid of the phase space. The grid points are transported with the flow of the transport equation for a time step and interpolated back periodically onto the initial grid. The method is then a mix of particle Lagrangian methods and Eulerian methods. The characteristics can be solved forward or backward in time leading to the Forward Semi-Lagrangian (FSL) or Backward Semi-Lagrangian (BSL) schemes. Conservative schemes based on this idea can be developed and are called Conservative Semi-Lagrangian (CSL).

GYSELA is a 5D full gyrokinetic code based on a classical backward semi-Lagrangian scheme (BSL) [31] for the simulation of core turbulence that has been developed at CEA Cadarache in collaboration with our team [28].

More recently, we have started to apply the semi-Lagrangian methods to more general kinetic equations. Indeed, most of the conservation laws of physics can be represented by a kinetic model with a small set of velocities and relaxation source terms [4]. Compressible fluids or MHD equations have such representations. Semi-Lagrangian methods then become a very appealing and efficient approach for solving these equations.

3.1.3. PIC methods

Historically PIC methods have been very popular for solving the Vlasov equations. They allow solving the equations in the phase space at a relatively low cost. The main disadvantage of this approach is that, due to its random aspect, it produces an important numerical noise that has to be controlled in some way, for instance by regularizations of the particles, or by divergence correction techniques in the Maxwell solver. We have a long-standing experience in PIC methods and we started implementing them in Selalib. An important aspect is to adapt the method to new multicore computers. See the work by Crestetto and Helluy [25].

3.2. Fluid and reduced kinetic models for plasmas

As already said, kinetic plasmas computer simulations are very intensive, because of the gyrokinetic turbulence. In some situations, it is possible to make assumptions on the shape of the distribution function that simplify the model. We obtain in this way a family of fluid or reduced models.

Assuming that the distribution function has a Maxwellian shape, for instance, we obtain the MagnetoHydro-Dynamic (MHD) model. It is physically valid only in some parts of the tokamak (at the edges for instance). The fluid model is generally obtained from the hypothesis that the collisions between particles are strong.

But the reduction is not necessarily a consequence of collisional effects. Indeed, even without collisions, the plasma may still relax to an equilibrium state over sufficiently long time scales (Landau damping effect).

In the fluid or reduced-kinetic regions, the approximation of the distribution function could require fewer data while still achieving a good representation, even in the collisionless regime.

Therefore, a fluid or a reduced model is a model where the explicit dependency on the velocity variable is removed. In a more mathematical way, we consider that in some regions of the plasma, it is possible to exhibit a (preferably small) set of parameters α that allows us to describe the main properties of the plasma with a generalized "Maxwellian” M. Then

$$f(x,v,t)=M(\alpha(x,t),v).$$

In this case it is sufficient to solve for $\alpha(x,t)$. Generally, the vector α is the solution of a first order hyperbolic system.

Another way to reduce the model is to try to find an abstract kinetic representation with an as small as possible set of kinetic velocities. The kinetic approach has then only a mathematical meaning. It allows solving very efficiently many equations of physics.

3.2.1. Numerical schemes

As previously indicated, an efficient method for solving the reduced models is the Discontinuous Galerkin (DG) approach. It is possible to make it of arbitrary order. It requires limiters when it is applied to nonlinear PDEs occurring for instance in fluid mechanics. But the reduced models that we intend to write are essentially linear. The nonlinearity is concentrated in a few coupling source terms.

In addition, this method, when written in a special set of variables, called the entropy variables, has nice properties concerning the entropy dissipation of the model. It opens the door to constructing numerical schemes with good conservation properties and no entropy dissipation, as already used for other systems of PDEs [32], [24], [30], [29].

3.2.2. Matrix-free implicit schemes

In tokamaks, the reduced model generally involves many time scales. Among these time scales, many of them, associated to the fastest waves, are not relevant. In order to filter them out, it is necessary to adopt implicit solvers in time. When the reduced model is based on a kinetic interpretation, it is possible to construct implicit schemes that do not impose solving costly linear systems. In addition the resulting solver is stable even at a very high CFL (Courant Friedrichs Lax) number.

3.3. Electromagnetic solvers

Precise resolution of the electromagnetic fields is essential for proper plasma simulation. Thus it is important to use efficient solvers for the Maxwell systems and its asymptotics: Poisson equation and magnetostatics.

The proper coupling of the electromagnetic solver with the Vlasov solver is also crucial for ensuring conservation properties and stability of the simulation.

Finally, plasma physics implies very different time scales. It is thus very important to develop implicit Maxwell solvers and Asymptotic Preserving (AP) schemes in order to obtain good behavior on long time scales.

3.3.1. Coupling

The coupling of the Maxwell equations to the Vlasov solver requires some precautions. The most important one is to control the charge conservation errors, which are related to the divergence conditions on the electric and magnetic fields. We will generally use divergence correction tools for hyperbolic systems presented for instance in [23] (and the references therein).

3.3.2. Implicit solvers

As already pointed out, in a tokamak, the plasma presents several different space and time scales. It is not possible in practice to solve the initial Vlasov-Maxwell model. It is first necessary to establish asymptotic models by letting some parameters (such as the Larmor frequency or the speed of light) tend to infinity. This is the case for the electromagnetic solver and this requires implementing implicit time solvers in order to efficiently capture the stationary state, the solution of the magnetic induction equation or the Poisson equation.

4. Application Domains

4.1. Controlled fusion and ITER

The search for alternative energy sources is a major issue for the future. Among others, controlled thermonuclear fusion in a hot hydrogen plasma is a promising possibility. The principle is to confine the plasma in a toroidal chamber, called a tokamak, and to attain the necessary temperatures to sustain nuclear fusion reactions.The International Thermonuclear Experimental Reactor (ITER) is a tokamak being constructed in Cadarache, France. This was the result of a joint decision by an international consortium including the European Union, Canada, USA, Japan, Russia, South Korea, India and China. ITER is a huge project. As of today, the budget is estimated at 20 billion euros. The first plasma shot is planned for 2020 and the first deuterium-tritium operation for 2027. Many technical and conceptual difficulties have to be overcome before the actual exploitation of fusion energy. Consequently, much research has been carried out around magnetically confined fusion. Amongt hese studies, it is important to carry out computer simulations of the burning plasma. Thus, mathematicians and computer scientists are also needed in the design of ITER. The reliability and the precision of numerical simulations allow a better understanding of the physical phenomena and thus would lead to better designs. TONUS's main involvement is in such research. The required temperatures to attain fusion are very high, of the order of a hundred million degrees. Thus it is imperative to prevent the plasma from touching the tokamak inner walls. This confinement is obtained thanks to intense magnetic fields. The magnetic field is created by poloidal coils, which generate the toroidal component of the field. The toroidal plasma current also induces a poloidal component of the magnetic field that twists the magnetic field lines. The twisting is very important or the stability of the plasma. The idea goes back to research by Tamm and Sakharov, two Russian physicists, in the 50's. Other devices are essential for the proper operation of the tokamak: divertor for collecting the escaping particles, microwave heating for reaching higher temperatures, fuel injector for sustaining the fusionr eactions, toroidal coils for controlling instabilities, etc.

4.2. Other applications

The software and numerical methods that we develop can also be applied to other fields of physics or of engineering.

- For instance, we have a collaboration with the company AxesSim in Strasbourg for the development of efficient Discontinuous Galerkin (DG) solvers on hybrid computers (CPU/GPU). The applications are electro-magnetic simulations for the conception of antennas, electronic devices or aircraft electromagnetic compatibility.

- The acoustic conception of large rooms requires huge numerical simulations. It is not always possible to solve the full wave equation and many reduced acoustic models have been developed. A popular model consists in considering "acoustic" particles moving at the speed of sound. The resulting Partial Differential Equation (PDE) is very similar to the Vlasov equation. The same modelling is used in radiation theory. We have started to work on the reduction of the acoustic particles model and realized that our reduction approach perfectly applies to this situation. A PhD with CEREMA (Centre d'études et d'expertise sur les risques, l'environnement, la mobilité et l'aménagement) has started in October 2015 (PhD of Pierre Gerhard). The objective is to investigate the model reduction and to implement the resulting acoustic model in our DG solver.
- In September 2017, we started a collaboration with EDF Chatou (PhD of Lucie Quibel) on the modelling of multiphase fluids with complex equations of state. The goal is to simulate the high temperature liquid-vapor flow occurring in a nuclear plant. Among others, we will apply our recent kinetic method for designing efficient implicit schemes for this kind of flows.

5. Highlights of the Year

5.1. Highlights of the Year

Low Mach relaxation scheme
We designed a new relaxation scheme [16]-[18] for the Euler/shallow water equations in the low Mach regime. The scheme admits an uniform convergence and a close to uniform cost compare to the Mach number. Additionally the implicit part (the most complicated classically) is reduced at the maximum. This method is a good candidate for the MHD in Tokamak and the extension of the method for this problem is an ongoing work.

6. New Software and Platforms

6.1. CLAC

Conservation Laws Approximation on many Cores

SCIENTIFIC DESCRIPTION: It is clear now that future computers will be made of a collection of thousands of interconnected multicore processors. Globally it appears as a classical distributed memory MIMD machine. But at a lower level, each of the multicore processors is itself made of a shared memory MIMD unit (a few classical CPU cores) and a SIMD unit (a GPU). When designing new algorithms, it is important to adapt them to this kind of architecture. Our philosophy will be to program our algorithms in such a way that they can be run efficiently on this kind of computers. Practically, we will use the MPI library for managing the coarse grain parallelism, while the OpenCL library will efficiently operate the fine grain parallelism.

We have invested for several years until now into scientific computing on GPUs, using the open standard OpenCL (Open Computing Language). We were recently awarded a prize in the international AMD OpenCL innovation challenge thanks to an OpenCL two-dimensional Vlasov-Maxwell solver that fully runs on a GPU. OpenCL is a very interesting tool because it is an open standard now available on almost all brands of multicore processors and GPUs. The same parallel program can run on a GPU or a multicore processor without modification.

Because of the envisaged applications of CLAC, which may be either academic or commercial, it is necessary to conceive a modular framework. The heart of the library is made of generic parallel algorithms for solving conservation laws. The parallelism can be both fine-grained (oriented towards GPUs and multicore processors) and coarse-grained (oriented towards GPU clusters). The separate modules allow managing the meshes and some specific applications. In this way, it is possible to isolate parts that should be protected for trade secret reasons.

FUNCTIONAL DESCRIPTION: CLAC is a generic Discontinuous Galerkin solver, written in C/C++, based on the OpenCL and MPI frameworks.

- Partner: AxesSim
- Contact: Philippe Helluy
- URL: http://clac.gforge.inria.fr/

6.2. Selalib

SEmi-LAgrangian LIBrary

KEYWORDS: Plasma physics - Semilagrangian method - Parallel computing - Plasma turbulence

SCIENTIFIC DESCRIPTION: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.

FUNCTIONAL DESCRIPTION: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

- Partners: Max Planck Insitute - Garching - Université de Strasbourg
- Contact: Philippe Helluy
- URL: http://selalib.gforge.inria.fr/

6.3. SCHNAPS

Solver for Conservative Hyperbolic Nonlinear Applications for PlasmaS

KEYWORDS: Discontinuous Galerkin - StarPU - Kinetic scheme

FUNCTIONAL DESCRIPTION: Generic systems of conservation laws. Specific models: fluids, Maxwell, Vlasov, acoustics (with kinetic representation). Multitasking with StarPU. Explicit solvers (RK2, RK3, RK4): accelerated with OpenCL Implicit solvers: through kinetic representations and palindromic time integration.

- Participants: Philippe Helluy, Matthieu Boileau and Bérenger Bramas
- Contact: Philippe Helluy
- URL: http://schnaps.gforge.inria.fr/

6.4. Slappy

KEYWORDS: Python - Opencl

FUNCTIONAL DESCRIPTION: The code Slappy solves the advection equations on multi-patch and non-conform complex geometries with the Semi-Lagrangian method. Using this we can also treat some hyperbolic/parabolic PDE with the Approximate BGK method which, allows to write a PDE as a transport plus a local relaxation step. The code is written in PyOpcenCL and can be used on CPU/GPU.

- Contact: Emmanuel Franck

6.5. Patapon

Parallel Task in Python

KEYWORDS: Python - Parallel computing - High order time schemes

FUNCTIONAL DESCRIPTION: Patapon is a code in PyOpenCL which allows to solve PDE like MHD using the vectorial Lattice Boltzmann method on Cartesian grids.

- Participant: Philippe Helluy
- Contact: Philippe Helluy

6.6. tofu

Tomography for Fusion

KEYWORDS: 3D - Data visualization - Visualization - Magnetic fusion - Tomography - Diagnostics - Plasma physics - Ray-tracing - Python

FUNCTIONAL DESCRIPTION: tofu aims at providing the fusion and plasma community with an object-oriented, transparent and documented tool for designing tomography diagnostics, computing synthetic signal (direct problem) as well as tomographic inversions (inverse problem). It gives access to a full 3D description of the diagnostic geometry, thus reducing the impact of geometrical approximations on the direct and, most importantly, on the inverse problem.

RELEASE FUNCTIONAL DESCRIPTION: Python 2.7 is not supported anymore Python 3.6 and 3.7 are supported Several changes to try and make installation easier (on clusters, windows, mac....) and less verbose for users More explicit names for default saved configurations Major bug fix in one of the methods for computing synthetic signal Minor bug fixes in interactive figures Minor bug fixes in Plasma2D interpolation New configuration (ITER) available First version of a class handling 2D XRay bragg spectrometers First tools for magnetic field line tracing available on WEST Better documentation, more ressources More informative error messages extra tools for computing LOS length, closest point to magnetic axis... Better PEP8 compliance

- Partner: CEA
- Contact: Laura Mendoza
- URL: https://github.com/ToFuProject/tofu

7. New Results

7.1. Relaxation method for Guiding-Center equation

Participants: E. Franck, R. Helie, L. Navoret, P. Helluy.
In previous years, implicit kinetic relaxation methods have been developed to treat conservation laws without CFL and without a non-trivial matrix to reverse [6]-[4]. We have started to apply this method with a spectral discretization for transport equations such as the guiding-center equation (a non constant advection equation coupled with elliptic problem used in plasma physics). The scheme obtained has a very high order of convergence for an instability test case and is very simple to implement. We have also investigated the different kinetic relaxation representations. However, they suffer from inaccuracy at the boundaries. We have proposed a new approach in 1D [8] to analyse this behaviour and a new way to apply boundary conditions to ensure they are compatible both with the approximated system and its kinetic approximation. Extending this approach to higher dimensions is one of the objectives of the thesis of Romane Helie.

7.2. Relaxation method for transport in Tokamak

Participants: M. Boileau, P. Helluy, B. Bramas (Inria Camus).
To apply the relaxation method in a Tokamak context, we have developed a code called Chukrut (in Schnaps) that can handle kinetic relaxation models in Tokamak geometry [15]-[17]-[13]. In the poloidal direction the code uses an unstructured Discontinuous Galerkin solver which solves the transport equation (the main ingredient of the kinetic relaxation method) by using the scheduling graph linked to the upwind scheme. In the toroidal direction we use an exact solver on uniform grids (which will be replaced by a semi-Lagrangian solver). The algorithm is parallelised in the poloidal plane by a task-based OpenMP implementation and in the toroidal direction by MPI parallelism.

7.3. Relaxation method for Euler/MHD in low-Mach regime

Participants: E. Franck, L. Navoret, F. Bouchut (Marne la Vallée university).
Previously, we have proposed implicit relaxation methods for fluid models that allow us to reverse a simple system. However, previous methods [5] were not very effective in the multi-scale regimes of interest. We therefore proposed a semi-implicit scheme based on a dynamic splitting and a relaxation of fast waves only. The scheme was first applied to the Euler equations in low Mach regime. The scheme is stable and accurate regardless of the Mach number. We have successfully applied the method for the equilibria of the Shallow Water equations. Since last summer we have begun the extension for the 1D MHD with and without dispersive effects. The first results show that we obtain a similar method compared to the Euler case with acceptable stability conditions as for the Euler equations.

7.4. Reduced model for the Scrape-Off Layer

Participants: L. Navoret, M. Mehrenberger, P. Ghendrih (CEA Cadarache)
In this work, we consider a one-dimensional model for describing the two-species plasma dynamics in the scrape-off layer. This region is defined as the transition between the core of the plasma and the edge and is located around the first non-closed magnetic field line. The electron and ion distribution functions satisfy a Vlasov-Poisson system with source and absorption terms and a non-homogeneous equilibrium is expected to develop. A high-order semi-Lagrangian scheme has been implemented to correctly capture such a dynamics.

7.5. Recurrence phenomenon for finite element grid based Vlasov solver

Participants: L. Navoret, M. Mehrenberger, N. Pham
We have improved our previous (last year) result concerning the recurrence phenomenon by providing a complete proof of the asymptotic behaviour of the correlation function. Indeed, we prove that, in the fine grid limit, the correlation function of the density exactly concentrates at multiple times of the recurrence time. This thus fully confirms the fact the amplitude of the recurrence phenomenon is actually linked to the spectral accuracy of the velocity quadrature when computing the charge density at least for the linear transport equation.

7.6. Machine learning techniques for reduced model and stabilization

Participants: E. Franck, L. Navoret, V. Vigon (IRMA Strasbourg).
Just recently, we have begun to work on applications of machine learning techniques for the plasma simulation. This preliminary work is in the context of "Action exploratoire MALESI" and will really begin in 2020. The first point is the construction of a new closure for the fluid models using kinetic simulation as data. We have constructed 1D solvers for the Vlasov-Poisson equation with collisional operator and Compressible Navier-Stokes Poisson models. Comparing the models we observe that the classical Navier-Stokes closure is not sufficient when the Knudsen number is larger than 0.3-0.4. Currently we generate data using the Vlasov-Poisson code to train a neural-network for the closure. The second point is about the stabilization of the numerical method using CNN. We began a study to construct a Convulational Neural Network (CNN) to detect the Gibbs oscillations in the fluid simulations.

7.7. Asymptotic Preserving scheme for Vlasov-Maxwell to MHD

Participants: E. Franck, A. Crestetto (Nantes university), M. Badsi (Nantes university).
The MHD equation can be obtained by taking the limit of different small parameters of the bi-species Vlasov-Maxwell equations. Obtaining an "asymptotic preserving" scheme for the Vlasov equation (cost independent of the small parameters) is an important goal. Indeed, this type of scheme would allows us to construct coupling methods between MHD and Vlasov equations or to make simulations in various regimes to construct closures with data (see the previous point). During this year we have written a scheme able to treat the "quasi-neutral" and "mass-less" limits between the two-species Euler-Maxwell equations and the MHD model. The scheme is partially validated. We will finish the validation and add the collisional limit between Vlasov-Maxwell and Euler-Maxwell equations.

7.8. Optimal control for population dynamics

Participants: Y. Privat, L. Almeida (Sorbonne University), M. Duprez (Dauphine University) and N. Vauchelet (Paris 13 University).
Particular attention is being paid to the transmission of dengue fever, an arbovirus transmitted to humans by mosquitoes [3]-[2]. There is no vaccine to immunize a population. It has been observed that when a mosquito population was infected with the Wolbachia virus, they stopped transmitting the disease. In addition, the virus is transmitted from mother to child and is characterized by cytoplasmic incompatibility (no possible crosses between infected males and healthy females). On the other hand, infected mosquitoes have a reduced lifespan and fertility. Mathematically, this situation can be modelled (in a simplified way) using a controlled reaction-diffusion system. The control term represents the strategy of releasing (time-space) mosquitoes infected by Wolbachia. The practical questions that arise and that we wish to address are:

- how to carry out these releases to ensure the invasion?
- how to optimize the domain and form of releases?

Preliminary work has made it possible to determine a plausible temporal control strategy.

7.9. Observability for wave equation and high frequency behavior

Participants: Y. Privat, E. Humbert (Tours University) et E. Trélat (Sorbonne University).
We have determined the asymptotic in time of the observability constant in closed manifolds. In particular, we have proved that this limit can be represented as the minimum of two quantities: one purely spectral and another called the geometric quantity representing the limit of the average time spent by geodesics within the observation domain [19]-[19].

7.10. Developement of a Python library for tomography diagnostics

Participants: L. Mendoza
In the tofu code, a big component of both the direct and inverse solvers is the integration module. During the 2019 project it was developed and accelerated. Special attention was brought to memory optimization. Core functions for the inversions routines were developed and parallelized using OpenMP. The number of users and developers of the library has significantly increased in the last year (collaborators in CEA cadarache, ITER, CEA saclay, IPP Garching, etc.) so one of the main objectives was to better the continuous integration and documentation of the code: more unitary and simulation tests have been implemented, an online Web site with the documentation has been added, the library can be used on more platforms (windows, mac os x, and linux), and more fusion devices are now available (West, ITER, JET, etc.).

7.11. Finite volume methods for complex hyperbolic systems

Participants: P. Helluy, L. Quibel (EDF)
This year we have developed a Lattice Boltzmann scheme able to treat really complex and tabuled EOS (Equation Of State) for compressible multiphase flows (two and three phases). This new scheme have been implemented in the PyOpenCL Patapon. Additionally, in order to perform realistic simulations of such situations, we have also proposed a code based on a model that can handle both the thermodynamical disequilibrium between liquid and vapor and complex equations. This code is based on a relaxation scheme which is the best compromise between accuracy and stability.

7.12. The study of domain walls in micromagnetism

Participants: C. Courtès, R. Côte (IRMA)

A ferromagnetic material consists of a succession of isolated subdomains (known as the magnetic domains) in which the magnetic moments are aligned and point in the same direction. The interface separating two magnetic domains is called the domain wall and corresponds to a localized area where the direction of the magnetization suddenly changes. Mathematically, those domain walls correspond to the minimizers of the well-known micromagnatics energy. The magnetic behavior of ferromagnetic materials is due to the arrangement of the magnetic domains and to the dynamics of their domains walls. That dynamic is governed by the nonlinear Landau-Lifshitz-Gilbert equation. We study numerically and theoretically the stability and the interaction of two domain walls. Depending on the initial topological configuration, two domain walls may collide to give rise to a persistent profile or annihilate both, which results in aligning all magnetization vectors of the nanowire in the same direction.

7.13. Maxwell solvers

Participants: P. Helluy, M. Houillon

In collaboration with the AxesSim company, we continue the development of our CLAC software devoted to electromagnetic simulations in biological environment. We have implemented a new wire model. We have also run computations on the new CNRS supercomputer: Jean Zay. We now routinely launch simulations on 64 V100 GPUs in parallel for performing parameter studies of various antennas near to the human body (we can for instance vary the humidity level of the skin).

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

We collaborate with EDF Chatou in the context of L. Quibel PhD. The objective is to design new Equations Of States (EOS) for the simulation of multiphase flows. The EOScannot be chosen arbitrarily if one wants to ensure the stability of the fluid model. We are also interested to apply our palindromic method for computing low-Mach liquid-vapor flows.

We are involved in a common project with the company AxesSim in Strasbourg. The objective is to help to the development of a commercial software for the numerical simulation of electromagnetic phenomena. The applications are directed towards antenna design and electromagnetic compatibility. This project was partly supported by DGA through "RAPID" funds. A CIFRE PhD has started in AxesSim on the same kinds of subjects in March 2015 (Bruno Weber). The new project is devoted to the use of runtime system in order to optimize DG solvers applied to electromagnetism [33]. The resulting software will be applied to the numerical simulation of connected devices for clothes or medicine. The project is supported by the "Banque Publique d'Investissement" (BPI) and coordinated by the Thales company.

9. Partnerships and Cooperations

9.1. Regional Initiatives

The thesis of Pierre Gerhard devoted to numerical simulation of room acoustics is supported by the Alsace-region. It is a joint project with CEREMA (Centre d'études et d'expertise sur les risques, l'environnement, lamobilité et l'aménagement) in Strasbourg.

9.2. National Initiatives

9.2.1. National projects

PEPS "initiative Jeunes" CNRS. E. Franck with A. Crestetto (leader), M. Badsi, "Asymptotic scheme for multiscale problems in Plasma".

PEPS "initiative Jeunes" CNRS. C. Courtès with R. Côte (IRMA), P. A. Hervieux (IPCMS), R. Ignat (IMT), G. Manfredi (IPCMS), "Study of the influence of the temperature and the external magnetic field on the magnetization reversal".

9.2.2. HPC resources

Big Challenge GENCI: Simulation of electromagnetic interaction between connected objects and the human body. We solve the 3D Maxwell equations to compute the antena emission Bluetooth Low Energy (BLE) close to the body. The main goal is to scale the computation on the new supercomputer Jean Zay to treat a realistic test case.

9.3. European Initiatives

9.3.1. FP7 & H2020 Projects

Eurofusion project MAGYK, *Mathematics and Algorithms for GYrokinetic and Kinetic models* (2019-2021), led by E. Sonnendrucker.
Participants: L. Navoret

Eurofusion project *Strengthening the non-linear MHD code JOREK for application to key questions of the fusion roadmap* (2019-2021), led by M. Hoelzl.
Participants: E. Franck

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Member of the Organizing Committees

Philippe Helluy: Workshop on Compressible Multiphase Flows : Derivation, closure laws, thermodynamics, IRMA, May 2019.

Yannick Privat with Raphaël Côte and Thomas Duyckaerts: Workshop Control and dynamics of PDEs, Strasbourg, October 2019.

10.1.2. Journal

10.1.2.1. Member of the Editorial Boards

Yannick Privat is a member of the editorial boards of:

- AIMS Applied Mathematics,
- Evolution Equations and Control Theory.

Philippe Helluy is member of editorial board in:

- Computational and Applied Mathematics,
- International Journal of Finite Volume.

10.1.2.2. Reviewer - Reviewing Activities

- Emmanuel Franck was a reviewer for:
  - Numerical Methods for Partial Differential Equations.
- Yannick Privat was a reviewer for:

  - Applied Math. and Optimization,
  - Esaim Control Optimization and Calculust of Variations,
  - Discrete and Continuous Dynamical Systems,
  - Esaim Proceeding,
  - Journal de Mathématiques Pures et Appliquées,
  - Journal of Math. Biology,
  - Proceedings of the Royal Society A,
  - SIAM Journal on Control and Optimization,
  - Zeitschrift für angewandte Mathematik und Physik.
- Philippe Helluy was a reviewer for:
  - Journal of Computational Physics,
  - Computers and Fluids,
  - Numerical Methods for Partial Differential Equations,
  - European Journal of Mechanics / B Fluids.
- Matthieu Boileau was a reviewer for:
  - Proceedings of the Combustion Institutes.

10.1.3. Invited Talks and participations to conference as speaker

Yannick Privat:

- New Trends in PDE-Constrained Optimization, Linz, Austria. October 2019.
- Shape optimization and application days, Ecole Polytechnique, France. October 2019.
- "Groupement Euro-Maghrébin de Mathématiques et leurs Interactions". Madrid, Spain. November 2019.
- Dynamics, Equations and Applications (organizer of a mini-symposium), Cracovia, Poland. September 2019.

Emmanuel Franck:

- ICIAM 2019 (organizer mini-symposium), Valencia, Spain. July 2019.
- Workshop Multi-scale, Inria Nancy, June 2019.
- Jorek Meeting 2019, Munich, May 2019.

Laurent Navoret:

- ICIAM 2019 (organizer of a mini-symposium), Valencia, Spain. July 2019.

P. Helluy:

- ICNAAM 2019, Rhodes, September 2019
- NUMHYP, Malaga, June 2019.
- NUMKIN, Garching, Ocotober 2019.

Laura Mendoza:

- ICIAM 2019, Valencia, Spain. July 2019.
- Iter codeCamp, Cadarache, April 2019.
- EuroScipy 2019, Bilbao, September 2019.
- PyConFr, Bordeaux, November 2019.

Matthieu Boileau:

- Computation and learning days. April 2019. Lyon.
- Workshop Compressible multiphase flows, Strasbourg 2019.

10.1.4. Research Administration

Philippe Helluy:

- Director of the IRMA mathematics institute.

Mickael Gutnic:

- Director of the mathematics department (IRMA) in Strasbourg university.

Yannick Privat:

- Member of CNU section 26,
- Member of expert committee at IRMA Strasbourg

Matthieu Boileau:

- Manager of the "GDR calcul",
- Co-Manager of the network "metier calcul",
- Member of the "Commission de la Recherche du Conseil Académique de l'Université de Strasbourg"

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence: E. Franck, "Informatics S6", 35h, L3, Strasbourg University, France.

Master: Y. Privat, "Optimal control", 64h, M2, Strasbourg University, France.

Master: Y. Privat, "Optimization", 64h, M1, Strasbourg University, France.

Licence: Y. Privat, "Nonlinear Optimization", 52h, L3, Strasbourg University, France.

Agrégation: Y. Privat, "Text study", 30h. Strasbourg University, France

Licence: C. Courtès, "Scientific computing", 37h, L3, Strasbourg University, France.

Licence: C. Courtès, "Numerical analysis", 34h, L3, Strasbourg University, France.

Licence: C. Courtès, "Numerical analysis", 18h, L2, Strasbourg Unversity, France.

Licence: C. Courtès, "Multiple variables function", 10h, L2, Strasbourg Unversity, France.

Licence: L. Navoret, "Nonlinear Optimization", 18h, L3, Strasbourg University, France.

Licence: L. Navoret, "Statistics for biologists", 21.5h, L2, Strasbourg University, France.

Master: L. Navoret, "Scientific computing", 35h, M1, Strasbourg University, France.

Master: L. Navoret, "Numerical methods for PDEs", 30h, M1, Strasbourg University, France.

Master: L. Navoret, "Optimization", 12h, M1, Strasbourg University, France.

Master (physics): L. Navoret, "Numerical resolution techniques for engineering", 22h, M1, Strasbourg University, France.

Master (Agregation): L. Navoret, "Scientific computing", 50h, M2, Strasbourg University, France.

Master (cell physics): L. Navoret, "Basics in mathematics", 24h, M2, Strasbourg University, France.

Licence, P. Helluy, "Numerical analysis", 17h, L3, Strasbourg University, France.

Master, P. Helluy, "Scientific computing", 20h, Préparation agrégation, Strasbourg University, France.

Master, P. Helluy, "Scientific computing", 28h, M1, Strasbourg University, France.

Master, P. Helluy, "Hyperbolic systems", 35h, M2, Strasbourg University, France.

Licence, M. Boileau, "informatic", 10h, L3, Strasbourg university, France.

Master, M. Boileau, "data sciences", 20h , M2, Strasbourg University, France.

Master, M. Boileau, "Parallel computing", 20h, M1, Strasbourg University, France.

Master, M. Boileau, "Python for science", 36h, Strasbourg University, France.

Other, M. Boileau, "Basic for python", 14h, Urfist (Unité Régionale de Formation à l'Information Scientifique et Technique), France.

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10.2.2. Supervision

PhD in progress: G. Mestdagh, "Suivi de tumeur en temps réel par des méthodes d'optimisation", Strasbourg university. Beginning: September 2019. Y. Privat, S. Cotin.

PhD in progress: J. B. Arnau, "Stratégie de contrôle d'une population de moustiques pour la lutte contre les arbovirus". Beginning: September 2019. Y. Privat, Luis Almeida.

PhD in progress: A. Courtais, "Contrôle optimal de contacteurs à lit fixe par fabrication additive". Beginning: September 2017. Y. Privat, F. Lesage and A. Latifi.

PhD in progress: Idriss Mazari, "Répartition des ressources dans un enclos optimisant la survie des populations". Beginning: September 2017. Y. Privat, G. Nadin.

PhD in progress: Alexandre Delyon, "Étude de la géométrie des œufs de certains branchiopodes". Beginning: September 2016. Y. Privat, A. Henrot.

PhD in progress: Mustafa Gaja, "Compatible Finite Elements for Wave and Fluid Models: Application to Plasma Physics". Beginning: October 2015. E. Franck, E. Sonnendrücker (main supervisor).

PhD in progress: Pierre Gerhard, "Numerical methods for kinetic models. Application to building acoustic". Beginning: Ocotober 2015. P. Helluy (main supervisor), L. Navoret.

PhD in progress: Romane Helie, "Relaxation methods for kinetic models in plasma physics". Beginning: October 2019. P. Helluy (main supervisor), L. Navoret, E. Franck.

PhD in progress: Marie Houillon: "Modeling of thin wires in electromagnetic software", Advisors:Philippe Helluy and Laurent Navoret, from October 2017, Labex Irmia support.

PhD in progress: Lucie Quibel (CIFRE support): in collaboration with EDF Chatou, from October2017, Advisor: Philippe Helluy.

PhD: Maxime Schmitt: "Optimization of scientific software with arbitrary mesh refinement", Defense: September 2019, Advisors: Philippe Helluy and Cédric Bastoul (CAMUS team). Labex Irmia support.

10.2.3. Juries

Y. Privat was member of jury of the PhD committee of F. Feppon, Ecole Polytechnique.

Y. Privat was member of jury of the PhD committee of A. Rebei. Paris Sciences et Lettres university.

Y. Privat was reviewer of jury of the PhD committee of N. Lebbe. Grenoble university.

P. Helluy was reviewer of jury of the PhD committee of S. Fornet. Toulouse, ISAE.

P. Helluy was reviewer of jury of the PhD committee of M. Kadiri. Normandie university.

P. Helluy was member of jury of the PhD committee of S. Bulteau. Nantes university.

P.Helluy was member of jury of the PhD committee of P. A. Giorgi. Aix-Marseille university.

10.3. Popularization

L. Navoret were allowed to participate in the "fete de la science"

11. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

[1] S. A. HIRSTOAGA. *Design and performant implementation of numerical methods for multiscale problems in plasma physics*, Université de Strasbourg, IRMA UMR 7501, April 2019, Habilitation à diriger des recherches, https://tel.archives-ouvertes.fr/tel-02081304

Articles in International Peer-Reviewed Journals

[2] L. Almeida, M. Duprez, Y. Privat, N. Vauchelet. *Mosquito population control strategies for fighting against arboviruses*, in "Mathematical Biosciences and Engineering", 2019, vol. 16, n^{o} 6, pp. 6274-6297, https://arxiv.org/abs/1901.05688 [*DOI :* 10.3934/mbe.2019313], https://hal.archives-ouvertes.fr/hal-01984426

[3] L. Almeida, Y. Privat, M. Strugarek, N. Vauchelet. *Optimal releases for population replacement strategies, application to Wolbachia*, in "SIAM Journal on Mathematical Analysis", 2019, vol. 51, n^{o} 4, pp. 3170–3194, https://arxiv.org/abs/1909.02727 [*DOI :* 10.1137/18M1189841], https://hal.archives-ouvertes.fr/hal-01807624

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[5] D. Coulette, E. Franck, P. Helluy, A. Ratnani, E. Sonnendrücker. *Implicit time schemes for compressible fluid models based on relaxation methods*, in "Computers and Fluids", June 2019, vol. 188, pp. 70-85 [*DOI :* 10.1016/j.compfluid.2019.05.009], https://hal.archives-ouvertes.fr/hal-01514593

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Scientific Books (or Scientific Book chapters)

[13] *Linear stability of a vectorial kinetic relaxation scheme with a central velocity*, HYP2018 proceedings, 2019, forthcoming, https://hal.archives-ouvertes.fr/hal-01970499

Other Publications

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