

*Inria*

IN PARTNERSHIP WITH:  
**CNRS**

**Ecole nationale supérieure des  
techniques avancées**

Activity Report 2019

## **Project-Team POEMS**

Wave propagation: mathematical analysis and  
simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

RESEARCH CENTER  
**Saclay - Île-de-France**

THEME  
**Numerical schemes and simulations**



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# Project-Team POEMS

*Creation of the Project-Team: 2019 November 01*

## **Keywords:**

### **Computer Science and Digital Science:**

- A6. - Modeling, simulation and control
- A6.1. - Methods in mathematical modeling
  - A6.1.1. - Continuous Modeling (PDE, ODE)
  - A6.1.2. - Stochastic Modeling
  - A6.1.4. - Multiscale modeling
  - A6.1.5. - Multiphysics modeling
  - A6.1.6. - Fractal Modeling
- A6.2. - Scientific computing, Numerical Analysis & Optimization
  - A6.2.1. - Numerical analysis of PDE and ODE
  - A6.2.2. - Numerical probability
  - A6.2.3. - Probabilistic methods
  - A6.2.7. - High performance computing
- A6.3.1. - Inverse problems
- A6.3.4. - Model reduction
- A6.5.1. - Solid mechanics
- A6.5.2. - Fluid mechanics
- A6.5.4. - Waves

### **Other Research Topics and Application Domains:**

- B2.6. - Biological and medical imaging
- B3.3. - Geosciences
  - B3.3.1. - Earth and subsoil
- B3.4. - Risks
  - B3.4.1. - Natural risks
  - B3.4.2. - Industrial risks and waste
- B5.3. - Nanotechnology
- B5.4. - Microelectronics
- B5.5. - Materials

## **1. Team, Visitors, External Collaborators**

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## **2. Overall Objectives**

### **2.1. Overall Objectives**

The propagation of waves is one of the most common physical phenomena in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) to the scales of the universe (electromagnetic waves, gravity waves) and of the atoms (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us. The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in various domains of physics and engineering sciences. The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas altogether justify a research project in applied mathematics and scientific computing devoted to this topic.

Nowadays, the numerical techniques for solving the basic academic problems are well mastered, and significant progress has been made during the last twenty years for handling problems closer to real applications. But several bottlenecks remain, among which one can mention the high-frequency problems for radar applications, the multiscale problems that arise for instance in nanotechnologies or the multi-physics couplings, like in aeroacoustics. Moreover, in the recent period, new challenges have emerged, related to new discoveries in physics (like negative index metamaterials) or to the fantastic development of information and communication techniques. For example, the growing development of increasingly connected objects (internet of things) and the forthcoming availability of autonomous vehicles depend crucially on electromagnetic waves, raising important issues about radar performance, sensor reliability, component miniaturization and electromagnetic compatibility. Generally, there are a lot of application domains which could benefit from advanced research on waves phenomena. Enhancing ultrasound-based methods for detection and imaging, which are already intensively used in e.g. medicine, could permit real-time health monitoring of aircrafts or nuclear plants. Guarding against seismic risks still requires considerable advances in the simulation of elastic waves in large and complex media. And many other applications motivating our research and our prospects could be added to this far-from-comprehensive list.

## 3. Research Program

### 3.1. Expertises

The activity of the team is oriented towards the design, the analysis and the numerical approximation of mathematical models for all types of problems involving wave propagation phenomena, in mechanics, physics and engineering sciences. Let us briefly describe our core business and current expertise, in order to clarify the new challenges that we want to address in the short and long terms.

Typically, our works are based on *boundary value problems* established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation of the hyperbolic type, whose prototype is the scalar wave equation, or the Helmholtz equation if time-periodic solutions are considered. More generally, we systematically consider both the transient problem, in the time domain, and the time-harmonic problem, in the frequency domain. Let us mention that, even if different waves share a lot of common properties, the transition from the scalar acoustic equation to the vectorial electromagnetism and elastodynamics systems raises a lot of mathematical and numerical difficulties, and requires a specific expertise.

A notable particularity of the problems that we consider is that they are generally set in *unbounded domains*: for instance, for radar applications, it is necessary to simulate the interaction of the electromagnetic waves with the airplane only, without any complex environment perturbing the wave phenomena. This raises an intense research activity, both from a theoretical and a numerical point of view. There exist several approaches which all consist in rewriting the problem (or an approximation of it) in a bounded domain, the new formulation being well-suited for classical mathematical and numerical techniques.

One class of methods consists in applying an appropriate condition on some boundary enclosing the zone of interest. In the frequency domain, one can use a non-local transparent condition, which can be expressed by a convolution with a Green function like in integral equation techniques, or by a modal decomposition when a separation of variables is applicable. But for explicit schemes in the time domain, local radiation conditions at a finite distance are generally preferred (constructed as local approximations at various orders of the exact non-local condition). A second class of methods consists in surrounding the computational domain by so called *Perfectly Matched absorbing Layers* (PML), which are very popular because they are easy to implement. POEMS members have provided several contributions to these two classes of methods for more than twenty-five years. Among them, one can mention the understanding of the instability of PMLs in anisotropic media and in dispersive media, the derivation of transparent boundary conditions in periodic media or the improvement of Fast Multipole techniques for elastodynamic integral equations.

In addition to more classical domains of applied mathematics that we are led to use (variational analysis and functional analysis, interpolation and approximation theory, linear algebra of large systems, etc...), we have acquired a deep expertise in *spectral theory*. Indeed, the analysis of wave phenomena is intimately linked to the study of some associated spectral problems. Acoustic resonance frequencies of a cavity correspond to the eigenvalues of a selfadjoint Laplacian operator, modal solutions in a waveguide correspond to a spectral problem set in the cross section. In these two examples, if the cavity or the cross-section is unbounded, a part of the spectrum is a continuum. Again, POEMS has produced several contributions in this field. In particular, a large number of significant results have been obtained for the existence or non-existence of guided modes in open waveguides and of trapped modes in infinite domains.

To end this far from exhaustive presentation of our main expertise domains, let us mention the *asymptotic techniques* with respect to some small scale appearing in the model: it can be the wavelength compared to the size of the scatterer, or on the contrary, the scale of the scatterer compared to the wavelength, it can be the scale of some microstructure in a composite material or the width of a thin layer or a thin tube. In each case, the objective, in order to avoid the use of costly meshes, is to derive effective simplified models. Our specificity here is that we can combine skills in physics, mathematics and numerics: in particular, we take care of the mathematical properties of the effective model, which are used to ensure the robustness of the numerical method, and also to derive error estimates with respect to the small parameter. There has been a lot of contributions of POEMS to this topic, going from the modeling of electromagnetic coatings to the justification of models for piezoelectric sensors. Let us mention that effective models for small scatterers and thin coatings have been used to improve imaging techniques that we are developing (topological gradient, time reversal or sampling techniques).

### 3.2. New domains

In order to consider more and more challenging problems (involving non-deterministic, large-scale and more realistic models), we decided recently to enlarge our domain of expertise in three directions.

Firstly, we want to reinforce our activity on *efficient solvers for large-scale wave propagation problems*. Since its inception, POEMS has frequently contributed to the development and the analysis of numerical methods that permit the fast solution of large-scale problems, such as high-order finite element methods, boundary elements methods and domain decomposition methods. Nevertheless, implementing these methods in parallel programming environments and dealing with large-scale benchmarks have generally not been done by the team. We want to continue our activities on these methods and, in a more comprehensive approach, we will incorporate modern algebraic strategies and high-performance computing (HPC) aspects in our methodology. In collaboration with academic or industrial partners, we would like to address industrial-scale benchmarks to assess the performance of our approaches. We believe that taking all these aspects into consideration will allow us to design more efficient wave-specific computational tools for large-scale simulations.

Secondly, up to now, *probabilistic methods* were outside the expertise of POEMS team, restricting us to deterministic approaches for wave propagation problems. We however firmly believe in the importance and usefulness of addressing uncertainty and randomness inherent to many propagation phenomena. Randomness may occur in the description of complex propagation media (for example in the modeling of ultrasound waves in concrete for the simulation of non-destructive testing experiments) or of data uncertainties. To quantify the effect of such uncertainties on the design, behavior, performance or reliability of many systems is then a natural goal in diverse fields of application.

Thirdly and lastly, we wish to develop and strengthen collaborations allowing a *closer interaction between our mathematical, modeling and computing activities and physical experiments*, where the latter may either provide reality checks on existing models or strongly affect the choice of modeling assumptions. Within our typical domain of activities, we can mention three areas for which such considerations are highly relevant. One is musical acoustics, where POEMS has made several well-recognized contributions dealing with the simulation of musical instruments. Another area is inverse problems, whose very purpose is to extract useful information from actual measurements with the help of (propagation) models. This is a core of our partnership with CEA on ultrasonic Non Destructive Testing. A third area is the modelling of effective (acoustic or



electromagnetic) metamaterials, where predictions based on homogenized models have to be confirmed by experiments.

## 4. Highlights of the Year

### 4.1. Highlights of the Year

- The POEMS EPI has been officially renewed by Inria on the 1st November 2019.
- Two permanent members of the team, S. Chaillat and S. Fliss, have successfully defended their habilitation theses.
- SACHEMS project led by CEA (where POEMS is involved) has been retained in the call of Ile-de-France Region SESAME 2019. Its purpose is to federate the research in the region in the field of SHM (Structural Health Monitoring), which consists of developing intelligent sensors aimed at detecting and characterizing directly defects in a structure (crack, corrosion, etc.). The SACHEMS project is part of a strategy for pooling equipment with the objective of creating an innovation platform for SHM methods.

## 5. New Software and Platforms

### 5.1. COFFEE

KEYWORDS: Numerical simulations - Wave propagation - Boundary element method

FUNCTIONAL DESCRIPTION: COFFEE is an adapted fast BEM solver to model acoustic and elastic wave propagation (full implementation in Fortran 90). The 3-D acoustic or elastodynamic equations are solved with the boundary element method accelerated by the multi-level fast multipole method or a hierarchical-matrices based representation of the system matrix. The fundamental solutions for the infinite space are used in this implementation. A boundary element-boundary element coupling strategy is also implemented so multi-region problems (strata inside a valley for example) can be solved. In order to accelerate the convergence of the iterative solver, various analytic or algebraic preconditioners are available. Finally, an anisotropic mesh adaptation strategy is used to further reduce the computational times.

- Contact: Stéphanie Chaillat

### 5.2. XLiFE++

KEYWORDS: Numerical simulations - Finite element modelling - Boundary element method

FUNCTIONAL DESCRIPTION: XLiFE++ is an FEM-BEM C++ code developed by POEMS laboratory and IRMAR laboratory, that can solve 1D/2D/3D, scalar/vector, transient/stationary/harmonic problems. Description: <https://uma.ensta-paristech.fr/soft/XLiFE++/>

- Contact: Eric Lunéville

## 6. New Results

### 6.1. Complex ordered and disordered media

There is a need of a better understanding of wave phenomena in complex media. From a physical point of view, a *complex medium* is typically a material where the propagation of the waves may be *anisotropic* and *dispersive*. These properties are generally the effect of a microstructure, that can be ordered (in e.g. photonic crystals), or disordered (light in the atmosphere, seismic waves). From a mathematical point of view, one can take into account exactly this microstructure or, at sufficiently low frequency, use effective models justified by the *homogenization theory*.

### 6.1.1. *Enriched homogenized model in presence of boundaries for time harmonic and time dependent wave equations*

**Participants:** Clément Bénéteau, Sonia Fliss.

We study the wave equation set in a periodic half-space when the period is small compared to the wave length. The classical homogenization theory enables to derive an effective model which provides an approximation of the solution. However it is well known that these models are not accurate near the boundaries. In this work, we propose an enriched asymptotic expansion which enables to derive high order effective models at order 1 and 2. Let us mention that the model of order 2 is particularly relevant when one is interested in the long time behaviour of the solution of the time-dependent wave equation. Indeed, it is well-known that the classical homogenized model does not capture the long time dispersion of the exact solution. In several works, homogenized models of order 2 are proposed for the wave equation in infinite domains. Dealing with boundaries and proposing boundary conditions for these models of order 2 were open questions. Our approach enables to propose appropriate and accurate boundary conditions for these models. This work is the fruit of a long time collaboration with Xavier Claeys (LJLL, Sorbonne University ) and a recent one with Timothée Pouchon (EPFL).

### 6.1.2. *Interface homogenization*

**Participant:** Jean-François Mercier.

In collaboration with Agnès Maurel from Institut Langevin and Kim Pham from the Department of Mechanics at ENSTA, we have developed interface effective models to describe acoustics and electromagnetic propagation through a scatterers array. The effective models are based on matched asymptotic expansions to account for the small thickness of the array. They consist of determined interface parameters involved in jump conditions for the fields.

1- In acoustics

- Perfect absorption using sparse arrays of Helmholtz resonators

Thanks to an effective model derived to describe a periodic arrangement of Helmholtz resonators, the influence of the spacing on the resonance has been inspected. The strength of the resonance is found enhanced when the array becomes sparser, which provides a degree of freedom to control the radiative damping of the array without affecting the losses within each resonator. It has been used to design a perfect absorbing wall.

- Scattering by arrays of open ended resonators

The previous study has been extended to cavities open at both ends. The effective model provides explicit expressions of the reflection and transmission coefficients, used to provide the relations required to produce zero reflection situation.

- Effective transmission conditions across a resonant bubbly metascreen

The extension to resonant obstacles has been considered with the study of the acoustic propagation through a thin bubbly screen. The analysis is conducted in the time domain and preserves the non linear response of the bubbles. It provides an effective model involving a jump of the normal velocity coupled to an equation of the Rayleigh-Plesset's type for the bubble radius.

2- In electromagnetism

- Perfect Brewster transmission by ultrathin perforated films

The scattering properties of an ultrathin perforated film, made of a material dielectric or perfectly conducting have been studied. Thanks to an asymptotic interface model, the Brewster incidence realizing perfect transmission is accurately described and is found to be significantly shifted from its classical value when the thickness of the film becomes subwavelength.

- Effective transmission conditions for an array of locally resonant inclusions

The previous study has been extended to resonant inclusions of the Mie type. Among the interface parameters involved in the effective model, one is frequency dependent and encapsulates the resonant behavior of the inclusions. Our effective model is validated by comparison with results of full wave calculations.

### **6.1.3. Wave equation in a weakly randomly perturbed periodic medium**

**Participants:** Sonia Fliss, Laure Giovangigli.

The aim of this work, which is at its first stage, is to construct numerical approximations of the solution of the wave equation in weakly randomly perturbed periodic media in order to propose transparent boundary conditions. We start by studying the effects of rare random perturbations of the medium. The perturbation is weak in the sense that it happens rarely but when it happens the correction is of the order of the initial coefficient. More precisely, we consider the solution of the time harmonic wave equation in a one-dimensional periodic medium, in which each period has a probability  $\eta$  to have its coefficients modified, independently of the other periods. We derive an asymptotic expansion of the distribution of the solution  $u_\eta$  with respect to  $\eta$  and illustrate the convergence with numerical simulations. We also exhibit and implement approximated transparent boundary conditions for such a medium. We then extend the results to more general rare random perturbations. Currently, we are studying other random perturbations of periodic media such as a deformation by a random diffeomorphism with a stationary gradient.

### **6.1.4. Guided modes in a hexagonal periodic graph-like domain: the zigzag and the armchair cases**

**Participant:** Sonia Fliss.

In this work, we study the wave propagation in hexagonal periodic media that are close to a graph domain. By using an asymptotic analysis, we exhibit situations where the introduction of lineic defects into the geometry of the domain leads to the appearance of guided modes and we show that the direction of the defect leads to very different properties of the guided modes. This work is done in collaboration with Bérangère Delourme (LAGA, Paris 13).

### **6.1.5. Stable perfectly matched layers for a class of anisotropic dispersive models**

**Participants:** Eliane Bécache, Maryna Kachanovska.

We consider wave propagation in 2D anisotropic dispersive media in an unbounded domain described by Maxwell's equations with an antisymmetric dielectric permittivity tensor and scalar magnetic permeability. Bounding the computational domain is required to obtain the solution. In order to do so, we use the perfectly matched layer (PML) technique. However, the PMLs exhibit instabilities connected to the presence of backward propagating waves. This work is dedicated to stabilizing the PMLs for this case.

### **6.1.6. Frequency domain wave propagation in anisotropic metamaterials**

**Participants:** Patrick Ciarlet, Maryna Kachanovska.

In this work we address the question of theoretical justification of problems arising in the wave propagation in hyperbolic metamaterials. Such phenomena are described by anisotropic, dispersive Maxwell equations, which, in the frequency domain, correspond to a problem that is hyperbolic for a range of frequencies. For a particular case of such materials (highly magnetized plasmas), we prove the well-posedness of the corresponding model in the free space, providing a suitable radiation condition, as well as study its regularity and demonstrate the limiting amplitude and limiting absorption principles.

### **6.1.7. On the analysis of perfectly matched layers for electromagnetic waves propagation in anisotropic media**

**Participants:** Eliane Bécache, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Maryna Kachanovska, Maria Kazakova.

This work consists of two parts. The first part is dedicated to the analysis of Cartesian Perfectly Matched Layers (PMLs) in the context of electromagnetic wave propagation in a 3D infinite anisotropic homogeneous medium with a diagonal dielectric tensor. Contrary to the 2D case some anisotropies lead to the existence of backward waves giving rise to instabilities of the PMLs in the time-domain and a lack of convergence in the frequency domain.

The second part examines the behaviour of the PMLs in the frequency domain in the case when in the time domain they give a rise to instabilities. This is the case e.g. for the 2D anisotropic wave equation. For this particular problem, we demonstrate that it is possible to choose the parameters of the PMLs (i.e. the configuration of the PML bounding box and the absorption parameter) to ensure the convergence of the PMLs in the frequency domain.

### **6.1.8. Maxwell's equations in presence of a conical tip with negative electromagnetic constants**

**Participants:** Anne-Sophie Bonnet-Ben Dhia, Mahran Rihani.

This work is done in collaboration with Lucas Chesnel from CMAP at Ecole Polytechnique. We are interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip which have infinite energy. In the 2D case of a wedge with critical electromagnetic constants, it has been proved for the equivalent scalar problems that well-posedness in the classical  $H^1$  framework is lost. Well-posedness can be recovered (in a non standard framework) by working in weighted Sobolev spaces and adding in the space the outgoing propagating singularity. We have shown how to provide such functional framework for 3D Maxwell's equations, when only the dielectric permittivity (but not the magnetic permeability) takes a critical value

### **6.1.9. Essential spectrum related to an interface with a negative material**

**Participants:** Christophe Hazard, Sandrine Paolantoni.

The studies carried out in recent years about the spectral effects of an interface between vacuum and a negative material (that is, a dispersive material whose electric permittivity and magnetic permeability become negative in some frequency range) have been continued in two directions. On the one hand, the previous theoretical studies only considered the non dissipative Drude model. We showed in particular that the interface is responsible for various resonance phenomena related to various components of an essential spectrum. We have extended these results to the so-called Lorentz model (dissipative or not). On the other hand, we have explored the numerical approximation of the spectrum of a cavity partially filled with a Drude material by considering a two-dimensional scalar problem. We have investigated the numerical simulation of the three resonance phenomena associated to the essential spectrum of the cavity.

### **6.1.10. Computation of plasmon resonances localized at corners using frequency-dependent complex scaling**

**Participants:** Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Florian Monteghetti.

A plasmonic device with a non-smooth boundary can exhibit strongly-oscillating surface waves whose phase velocities vanish as they reach the corners. This work investigates in the quasi-static limit the existence of corner resonances, which are analogous to scattering resonances in the sense that the local behavior at each corner plays the role of the behavior at infinity. Resonant contrasts are sought as eigenvalues of the transmission problem with complex scaling applied at corners.

### **6.1.11. Towards non-local interface models**

**Participant:** Patrick Ciarlet.

Collaboration with Juan Pablo Borthagaray (DMEL, Universidad de la República, Salto, Uruguay). Consider the equation  $\operatorname{div}(\sigma \nabla u) = f$  in  $\Omega$  (plus boundary conditions), where the diffusivity is piecewise constant, and equals  $\sigma_i$  in  $\Omega_i$  ( $i = \{1, 2\}$ ), with  $\overline{\Omega_1} \cup \overline{\Omega_2} = \overline{\Omega}$  and  $\Omega_1 \cap \Omega_2 = \emptyset$ . If  $\sigma_1$  and  $\sigma_2$  have different sign, well-posedness in  $H^1(\Omega)$  may not hold. This occurs when the ratio  $\sigma_2/\sigma_1$  belongs to the so-called *critical interval*. When the interface has a corner, we have observed that this critical interval is shrunk if one replaces the standard  $H^1$ -bilinear forms by corresponding  $H^s$ -forms ( $s \in (0, 1)$ ). However, the expense of computing the nonlocal interactions may be prohibitive in applications. Thus, our long term goal is to confine the non-local model to a neighborhood of the interface, while keeping the standard local model in the rest of the domain. A first step in this direction consists in considering the numerical solution of the fractional Laplacian of index  $s \in (1/2, 1)$  in a bounded domain  $\Omega$  with homogeneous Dirichlet boundary conditions. Its solution a priori belongs to the fractional order Sobolev space  $\tilde{H}^s(\Omega)$ . Under suitable assumptions on the data, its solution is also in  $H^1(\Omega)$ . In this case, if one uses the standard Lagrange finite element to discretize the problem, then both the exact and the computed solution belong to  $H^1(\Omega)$ . We show how to derive error estimates for the Lagrange finite element solutions on both quasi-uniform and graded meshes.

### 6.1.12. *Perturbed edge finite element method for the simulation of electromagnetic waves in magnetised plasmas*

**Participants:** Damien Chicaud, Patrick Ciarlet, Axel Modave.

Numerical simulation of electromagnetic waves in magnetised plasmas is a challenging topic. We address the finite element solution of a time-harmonic model. With the classical method, the variational formulation has a poor coercivity which leads to an ill-conditioned numerical system and numerical instabilities. We propose a perturbed formulation to improve the conditioning of the system. Promising preliminary numerical results have been obtained.

### 6.1.13. *Resonant wave problems in plasmas*

**Participant:** Patrick Ciarlet.

Collaboration with Martin Campos Pinto, Bruno Després and Anouk Nicolopoulos (LJLL, Sorbonne Université). The modelling of resonant waves in 2D plasma leads to the coupling of two degenerate elliptic equations. The model is set over two regions, and involves a smooth, sign-changing coefficient  $\alpha$ . The region where  $\{\alpha > 0\}$  is propagative, while the region where  $\{\alpha < 0\}$  is non propagative, and elliptic. The two models are coupled through the line  $\Sigma = \{\alpha = 0\}$ . Generically, it is an ill-posed problem, and additional information must be introduced to get a satisfactory treatment at  $\Sigma$ . We define the solution by relying on the limiting absorption principle (the coefficient  $\alpha$  is replaced by  $\alpha + i0^+$ ) in an adapted functional setting. This approach relies on the decomposition of the solution in a regular and a singular part, which originates at  $\Sigma$ , and on quasi-solutions. It yields a well-posed mixed variational formulation with coupling. After the design of explicit quasi-solutions, numerical experiments can be carried out, which illustrate the nice properties of this new tool.

## 6.2. **Towards realistic configurations : waveguides and fractals domains**

To simulate realistic wave problems, devices which raise specific difficulties concerning either the modeling, the mathematical analysis or the numerical simulation. We start with propagation in *waveguides*, that is a longtime research field within our team, which has acquired an international visibility in this context. We continue with a more recent topic, propagation in *fractal* domains, motivated by a medical application (the human lung).

### 6.2.1. *Transparent boundary conditions for periodic waveguides: analysis and extensions*

**Participants:** Sonia Fliss, Patrick Joly.

We consider the time harmonic wave equation in perturbed periodic waveguides. We justify rigorously the construction of the transparent boundary conditions based on Dirichlet-to-Neumann map and show that the problem with these transparent boundary conditions is of Fredholm type except for a countable set of frequencies. This allows to define and compute the physical solution of the problem. This approach can be applied to deal with junctions of different periodic closed waveguides. We want now to study the extension of the method to the diffraction by locally perturbed periodic layers, surfaces or halfspaces. This work is done in collaboration with Vincent Lescarret (LSS, Centrale Supélec).

### 6.2.2. *Invisible floating objects*

**Participant:** Mahran Rihani.

This work is done in collaboration with Lucas Chesnel from CMAP at Ecole Polytechnique. We consider a time-harmonic water waves problem in a 2D waveguide. The geometry is symmetric with respect to an axis orthogonal to the direction of propagation of waves. Moreover, the waveguide contains two floating obstacles separated by a distance  $L$ . We study the behaviours of the scattering coefficients as  $L$  goes to  $\infty$ . From this analysis, we exhibit situations of non reflectivity or perfect invisibility.

### 6.2.3. *A multi-trace integral equation on infinite boundaries when a global Green's function is not available*

**Participants:** Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Yohanes Tjandrawidjaja.

We are interested in time-harmonic scattering problems for configurations where the Green's function is not easily computable for the exterior domain, but different Green's functions are available in several unbounded subdomains covering the whole space. This arises typically for junctions of open waveguides. For a model problem, by using integral representations of the solution in each subdomain, we propose a formulation coupling the traces and the normal traces of the solution on infinite boundaries. The system of equations is shown to have a unique solution in the dissipative case.

### 6.2.4. *Error analysis for transparent boundary conditions in fractal trees*

**Participants:** Patrick Joly, Maryna Kachanovska.

This work is dedicated to an efficient resolution of the wave equation in fractal trees (with application to wave propagation in a human lung). Thanks to self-similarity, it is possible to avoid computing the solution at deeper levels of the tree by using transparent boundary conditions. The corresponding DtN operator is defined by a functional equation for its symbol. In the frequency domain. In this work, we analyse an approximate transparent condition, cf. Waves 2017, based on rational approximation of the symbol. The error and complexity analysis relies on Weyl-like estimates of eigenvalues of the weighted Laplacian and related eigenfunctions.

## 6.3. Direct and inverse methods for imaging and identification

Imaging and identification, when involved in a real-life context, are often based on wave propagation. This is due to the fact that a substantial part of the information contained in waves can propagate across long distances without significant attenuation. This activity is partly developed in the framework of a long-term partnership with a group of CEA-List in charge of the Non Destructive Testing (NDT) of industrial structures. The aim of NDT is to detect defects inside a structure by imposing some incident waves and measuring the scattered waves caused by the presence of such defects.

### 6.3.1. *The complex-scaled Halfspace Matching Method*

**Participants:** Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Sonia Fliss, Yohanes Tjandrawidjaja.

We are currently developing a method that we call the Half-Space Matching (HSM) method, to solve scattering problems in unbounded domains, when classical approaches are either not applicable or too expensive. This method is based on an explicit expression of the "outgoing" solution of the problem in half-spaces, by using Fourier, generalized Fourier or Floquet transforms when the background is respectively homogeneous (possibly anisotropic), stratified or periodic. The domain exterior to a bounded region enclosing the scatterers is recovered by a finite number of halfspaces (at least 3). The unknowns of the formulations are the restriction of the solution to the bounded region and the traces of the solution on the boundary of the halfspaces. The system of equations is derived by writing compatibility conditions between the different representations of the solution. Although the HSM method works in the non-dissipative case, the theoretical and the numerical analysis of the method has been done only in the dissipative case. In the present work, we propose, for the simple case of a homogeneous background, a new formulation of the method which is well-suited for the theoretical and numerical analysis of the non dissipative case. In the spirit of PMLs, the idea is to replace the system of equations on the traces by similar equations on exponentially decaying analytical extensions of the traces.

### 6.3.2. *Implicit-explicit scheme for elastodynamic equations in plates*

**Participants:** Sonia Fliss, Hajer Methenni.

Our objective is to provide an efficient simulation tool for the propagation of elastic waves in thin plates in the context of Guided Waves based Structural Health Monitoring. A naive discretization procedure based on a Leap-frog explicit scheme can be really costly because of the small thickness of the plate. By treating implicitly the operators corresponding to derivatives through the thickness, we show by a stability analysis that the time step is less restricted by the space discretization along the thickness. The price to pay is to solve at each iteration small independent linear systems, but this strategy offers an accurate and efficient discretization of the elastic fields in all dimensions. This method can be used to compute reference solutions and verify the validity of asymptotic models such as Reissner–Mindlin model and some extensions (since there exists no rigorous justifications for elastodynamic problems). Finally under some conditions on the mesh, our approach can be extended to plates with a smoothly varying thickness.

This work is done in collaboration with Sebastien Imperiale (Inria EPI M3DISIM) and Alexandre Imperiale (CEA-LIST).

### 6.3.3. *Forward and inverse scattering in Kirchhoff plates*

**Participants:** Laurent Bourgeois, Christophe Hazard.

A new activity has just started concerning forward and inverse scattering in thin plates governed by the simple Kirchhoff-Love model. The analysis is restricted to the purely bending case and the time-harmonic regime.

We have first considered a 2D strip, that is a waveguide which is unbounded in one direction and bounded in the other (transverse) direction. Two types of conditions on the boundary of the strip are addressed : either the strip is simply supported or the strip is clamped. The two boundary conditions are treated with two different methods. For the simply supported problem, the analysis is based on a result of Hilbert basis in the transverse section. For the clamped problem, this property does not hold. Instead we adopt the Kondratiev's approach, based on the use of the Fourier transform in the unbounded direction, together with techniques of weighted Sobolev spaces with detached asymptotics. After introducing radiation conditions, the corresponding scattering problems in the presence of a free obstacle are shown to be well-posed in the Fredholm sense. We also show that the solutions are the physical (outgoing) solutions in the sense of the limiting absorption principle. This is a joint work Lucas Chesnel, from Inria/DEFI.

We have then addressed the same kind of forward scattering problems for various impenetrable obstacles in an infinite plate. Considering four types of boundary conditions on the obstacle, well-posedness for those problems is proved with the help of a variational approach: (i) for any wave number  $k$  when the plate is clamped, simply supported or roller supported; (ii) for any  $k$  except a discrete set when the plate is free (this set is finite for convex obstacles). It is then natural to tackle the inverse problem of identifying impenetrable obstacles in a Kirchhoff-Love infinite plate from multistatic near-field data. The Linear Sampling Method is

introduced in this context. We firstly prove a uniqueness result for such an inverse problem. We secondly provide the classical theoretical foundation of the Linear Sampling Method. We lastly show the feasibility of the method with the help of numerical experiments. The inverse problem is a joint work with Arnaud Recoquillay, from CEA/LIST.

#### **6.3.4. About regularity and error estimates for the quasi-reversibility method**

**Participant:** Laurent Bourgeois.

This work is done on collaboration with Lucas Chesnel (EPC DEFI). We are interested in the classical ill-posed Cauchy problem for the Laplace equation. One method to approximate the solution associated with compatible data consists in considering a family of regularized well-posed problems depending on a small parameter  $\varepsilon > 0$ . In this context, in order to prove convergence of finite elements methods, it is necessary to get regularity results of the solutions to these regularized problems which hold uniformly in  $\varepsilon$ . In the present work, we obtain these results in smooth domains and in 2D polygonal geometries. In the presence of corners, due to the particular structure of the regularized problems, classical techniques *à la* Grisvard do not work and instead, we apply the Kondratiev approach. We describe the procedure in detail to keep track of the dependence in  $\varepsilon$  in all the estimates. The main originality of this study lies in the fact that the limit problem is ill-posed in any framework.

#### **6.3.5. Analysis of topological derivative as a means for qualitative identification**

**Participant:** Marc Bonnet.

This work is done on collaboration with Fioralba Cakoni, Rutgers University, USA. The concept of topological derivative (TD) has proved effective as a qualitative inversion tool for a wave-based identification of finite-sized objects. Although for the most part, this approach remains based on a heuristic interpretation of the TD, a first attempt toward its mathematical justification was done in Bellis et al. (*Inverse Problems* **29**:075012, 2013) for the case of isotropic media with far field data and inhomogeneous refraction index. This work extends the analysis there to the case of anisotropic scatterers and background with near field data. TD-based imaging functional is analyzed using a suitable factorization of the near fields. Our results include justification of sign heuristics for the TD in the isotropic case with jump in the main operator and for some cases of anisotropic media, as well as verifying its decaying property in the isotropic case with near field spherical measurements configuration situated far enough from the probing region.

#### **6.3.6. Asymptotic model for elastodynamic scattering by a small surface-breaking defect**

**Participant:** Marc Bonnet.

*This work is done in collaboration with Marc Deschamps and Eric Ducasse, I2M, Bordeaux.*

We establish a leading-order asymptotic model for the scattering of elastodynamic fields by small surface-breaking defects in elastic solids. The asymptotic form of the representation formula of the scattered field is written in terms of the elastodynamic Green's tensor, which is in fact available in semi-analytical form for some geometrical configurations that are of practical interest in ultrasonic NDT configurations. Preliminary numerical examples have been performed on cylindrical elastic pipes with small indentations on the outer surface.

#### **6.3.7. Shape optimization of stokesian peristaltic pumps using boundary integral methods**

**Participant:** Marc Bonnet.

This work is done in collaboration with with Ruowen Liu and Shrahan Veerapaneni, University of Michigan, USA.

This work develops a new boundary integral approach for finding optimal shapes of peristaltic pumps that transport a viscous fluid. Formulas for computing the shape derivatives of the standard cost functionals and constraints, expressed in boundary-only form, are derived. They involve evaluating physical variables (traction, pressure, etc.) on the boundary only. By employing these formulas in conjunction with a boundary integral approach for solving forward and adjoint problems, we completely avoid the issue of volume remeshing when updating the pump shape as the optimization proceeds. This leads to significant cost savings and we demonstrate the performance on several numerical examples.



## 6.4. Accelerated numerical solvers for large-scale wave problems

Fast solution procedures are of critical importance for industrial applications such as non-destructive testing, electromagnetic compatibility testing and seismic risk assessment. In these examples, the wavelength is very small in comparison to the characteristic length of the problems, which leads to extremely expensive numerical procedures if standard methods are used. To address the fast numerical solution of large-scale waves problems, we work at the same time on numerical methods, algorithmic issues and implementation strategies to speed up solvers.

### 6.4.1. *Non-overlapping Domain Decomposition Method (DDM) using non-local transmission operators for wave propagation problems.*

**Participants:** Patrick Joly, Emile Parolin.

The research in this direction was mainly concerned by the extension to the electromagnetic setting of the linear convergence theory of non-overlapping DDM that relies on non-local transmission operators. The principal task was to propose, analyse and implement some candidate non-local operators satisfying the assumptions of the theory. There were two main propositions:

- Integral operator for the electromagnetic setting: the operator is available in closed form and its structure lead naturally to a localizable form via truncation of the kernel to limit the effective computational cost while retaining its good properties. The construction of such an operator turned out to be somewhat difficult due to the particular functional setting of Maxwell's equations.
- DtN based non-local operator: the operator is computed by solving auxiliary coercive problems in the vicinity of the transmission interface. The computational cost remains moderate as the implementation no longer involve dense matrix blocks from the integral operators but rather lead to augmented sparse linear systems. Initially developed for the electromagnetic setting, the approach is appealing as it provided a unified formalism that can be applied both to Helmholtz and Maxwell equations and proved to be efficient in numerical experiments.

Another important research direction is created by the technical and theoretical difficulty posed by junction points, which are points where three or more sub-domains abut. Xavier Claeys recently proposed a method to deal with this specific issue, based on the multi-trace formalism, which led to a joint collaboration on the subject. The main idea is to perform a global exchange operation, on the whole skeleton, rather than a local point-to-point exchange. The preliminary numerical results recently obtained are promising.

### 6.4.2. *An efficient domain decomposition method with cross-point treatment for Helmholtz problems*

**Participant:** Axel Modave.

This is a collaboration with X. Antoine (IECL, Nancy), A. Royer (ULiège) and C. Geuzaine (ULiège). The parallel finite-element solution of large-scale time-harmonic scattering problems is addressed with a non-overlapping domain decomposition method (DDM). It is well known that the efficiency of this method strongly depends on the transmission condition enforced on the interfaces between the subdomains. Local conditions based on high-order absorbing boundary conditions (HABCs) are well suited for configurations without cross points (*where more than two subdomains meet*). In this work, we extend this approach to efficiently deal with cross points. Two-dimensional finite-element results are presented.

### 6.4.3. *Modelling the fluid-structure coupling caused by a far-field underwater explosion using a convolution quadrature based fast boundary element method.*

**Participants:** Marc Bonnet, Stéphanie Chaillat, Damien Mavaleix-Marchessoux.

This study is done in collaboration with Bruno Leblé (Naval Group). It aims at developing computational strategies for modelling the impact of a far-field underwater explosion shock wave on a structure, in deep water. An iterative fluid-structure coupling is developed to solve the problem. Two complementary methods are used: the Finite Element Method (FEM), that offers a wide range of tools to compute the structure response; and the Boundary Element Method (BEM), more suitable to deal with large surrounding fluid domains. We concentrate on developing (i) a fast transient BEM procedure and (ii) a transient FEM-BEM coupling algorithm. The fast transient BEM is based on a fast multipole-accelerated Laplace-domain BEM (implemented in the in-house code COFFEE), extended to the time domain by the Convolution Quadrature Method (CQM). In particular, using empirical approximations for the solution of integral problems involving large (complex) frequencies has been found to yield satisfactorily accurate solutions while saving significant amounts of computational work. The transient BEM-FEM coupling (under progress) will be based on a block-SOR iterative approach, for which a preliminary investigation shows the existence of relaxation parameters that ensure convergence.

#### 6.4.4. Asymptotic based methods for very high frequency problems.

**Participant:** Eric Lunéville.

This research is developed in collaboration with Marc Lenoir and Daniel Bouche (CEA).

It has recently been realized that the combination of integral and asymptotic methods was a remarkable and necessary tool to solve scattering problems, in the case where the frequency is high and the geometry must be finely taken into account.

In order to implement the high-frequency approximations that we are developing as part of these hybrid HF/BF methods, we have introduced new geometric tools into the XLiFE++ library, in particular splines and B-Splines approximations as well as parameterizations to access quantities such as curvature, curvilinear abscissa, etc. We have also started to interface the OpenCascad library to the XLiFE++ library, which will eventually allow us to manage more complex geometric situations (cylinder and sphere intersection for example). In parallel, we have completed the implementation of 2D HF approximations in the shadow-light transition zone based on the Fock function. Diffraction by a 2D corner is in progress.

## 7. Bilateral Contracts and Grants with Industry

### 7.1. Bilateral Contracts with Industry

- Contract and CIFRE PhD with Naval Group on *modelling the fluid-structure coupling caused by a far-field underwater explosion*  
Participants: M. Bonnet, S. Chaillat, D. Mavaleix-Marchessoux  
Start: 11/2017. End: 10/2020. Administrator: CNRS
- Contract and CIFRE PhD with Naval Group on *flow noise prediction*  
Participants: J-F Mercier, B. Cotté, N. Trafny  
Start: 04/2018. End: 03/2021. Administrator: ENSTA
- Contract and CIFRE PhD with CEA on *Modelling of thin layers of randomly distributed nanoparticles for electromagnetic waves* Participants: A. Boucart, S. Fliss, L. Giovangigli  
Start: 10/2019. End: 09/2022. Administrator: ENSTA

## 8. Partnerships and Cooperations

### 8.1. National Initiatives

#### 8.1.1. ANR

- ANR project NonlocalDD (*Non-local domain decomposition methods in electromagnetics*)  
Partners: Inria Alpines, Inria POEMS, Inria Magique 3D  
Start: 10/2015. End: 09/2020. Administrator: Inria  
Participants of POEMS: S. Chaillat, P. Joly  
Coordinator: X. Claeys (LJLL, EPI ALPINES)
- ANR project MODULATE (*Modeling lOng-perioD groUnd motions, and assessment of their effects on Large-scale infrAsTructurEs*)  
Partners: ENSTA (UME), Inria POEMS, CentraleSupélec, BRGM, GDS  
Start: 11/2018. End: 10/2021. Administrator: ENSTA  
Participant of POEMS: S. Chaillat  
Coordinator: K. Meza Fajardo (BRGM)

### 8.1.2. DGA

- Contracts between DGA and POEMS:
  - Contract on *boundary element methods and high-frequency problems*  
Participants: E. Lunéville, M. Lenoir, N. Kielbasiewicz.  
Start: 10/2018. End: 09/2021. Administrator: ENSTA  
*In partnership with F. Alouges and M. Aussal (CMAP, Ecole Polytechnique).*
- DGA provides partial funding for several PhD students:
  - C. Bénéteau on the *asymptotic analysis of time harmonic Maxwell equations in presence of metamaterials* (Start: 10/2017)
  - D. Chicaud on *domain decomposition methods for time-harmonic electromagnetic wave problems with complex media* (Start: 10/2018)

## 8.2. International Initiatives

### 8.2.1. Inria International Partners

#### 8.2.1.1. Informal International Partners

Juan Pablo Borthagaray (Universidad de la República, Uruguay)  
 Shravan Veerapaneni (Univ. of Michigan at Ann Arbor, USA)  
 Bojan Guzina (University of Minnesota, USA)  
 Jean-François Molinari (EPFL, Lausanne, Switzerland)  
 Fioralba Cakoni (University of Rutgers, USA)  
 Wilkins Aquino (Duke University, USA)  
 Bojan Guzina (University of Minnesota, USA)  
 Jorge Albella (University of Santiago de Compostela, Spain)  
 Carlos Perez Arancibia (Pontificia Universidad Católica, Chile)  
 Camille Carvalho (UC Merced, Merced, USA)  
 Simon Chandler Wilde (University of Reading, UK)  
 Mahadevan Ganesh (Colorado School of Mines, USA)  
 Christophe Geuzaine (Université de Liège, Belgium)  
 Marcus Grote (Universitaet Basel, Switzerland)  
 Moez Khenissi (Univesité de Sousse, Tunisia)  
 Sergei Nazarov (Saint-Petersburg University, Russia)  
 Karl-Mikael Perfekt (University of Reading, UK)  
 Jerónimo Rodríguez (University of Santiago de Compostela, Spain)

Ruben Rosales (MIT, USA)  
 Adrien Semin (TU Darmstadt, Germany)  
 Knut Sølna (University of California, Irvine, USA)  
 Catalin C. Turc (NJIT, NJ, USA)  
 Jun Zou (Chinese University of Hong Kong, HK)

## 8.3. International Research Visitors

### 8.3.1. Visits of International Scientists

- Mahadevan Ganesh (Colorado School of Mines, USA) — March 2019
- Carlos Jerez-Hanckes (Universidad Adolfo Ibanez, Chile) — Septembre 2019
- Shravan Veerapaneni (Univ. of Michigan at Ann Arbor, USA) — November 2019

## 9. Dissemination

### 9.1. Promoting Scientific Activities

#### 9.1.1. Scientific Events: Organisation

##### 9.1.1.1. General Chair, Scientific Chair

- Several members of POEMS team were members of the scientific committee of the WAVES conference, organized in Vienna in August 2019.
- J.-F. Mercier co-organized a summer school on "Wave propagation in complex and microstructured media" at Cargèse (Corsica), August 20th to 30th, 2019
- A.-S. Bonnet-Ben Dhia co-organized a workshop entitled "Advanced Theoretical and Numerical Methods for waves in structured Media" in Marseille in June 2019, in the framework of the GDR Ondes. They were about 90 attendees.

##### 9.1.1.2. Member of the Organizing Committees

- S. Chaillat is a co-animator of the topic "Modeling and simulation" of the GDR Ondes.
- J.-F. Mercier is a co-animator of the topic "Effective dynamics of microstructured media" of the GDR MecaWaves.

#### 9.1.2. Journal

##### 9.1.2.1. Member of the Editorial Boards

- A. S. Bonnet-Ben Dhia is an associate editor of SIAP (*SIAM Journal of Applied Mathematics*).
- M. Bonnet is an associate editor of *Engineering Analysis with Boundary Elements*, *Journal of Optimization Theory and Application* and *Journal of Integral Equations and Applications*. He is in the editorial board of *Inverse Problems* and *Computational Mechanics*.
- P. Ciarlet is an editor of ESAIM:M2AN (*Mathematical Modeling and Numerical Analysis*).
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.

##### 9.1.2.2. Reviewer - Reviewing Activities

The team members regularly review papers for many international journals.

### 9.1.3. Research Administration

- E. Lunéville was chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris until September 2019.
- A. S. Bonnet-Ben Dhia is deputy-chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris since October 2019.
- P. Ciarlet is coordinator of the *Mathematics in Computational Science and Engineering Program* of the Mathematics Hadamard Labex (LMH).
- E. Bécache is a deputy chair of the Doctoral School EDMH since December 2019.

## 9.2. Teaching - Supervision - Juries

### 9.2.1. Teaching

The permanent members of POEMS are involved in the engineering program at ENSTA Paris, the master program "*Analyse, Modélisation et Simulation*" (AMS) and the master program "*Modélisation et Simulation en Mécanique des Structures et Systèmes Couplés*" (MS2SC) of Université Paris-Saclay.

Eliane Bécache

- *Fonctions de variable complexe*, ENSTA (1st year)
- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
- *La méthode des éléments finis*, ENSTA (2nd year) and Master AMS (M1)
- *Analyse et approximation par éléments finis d'EDP*, ENSTA (2nd year) and Master AMS (M1)
- *Résolution des problèmes de diffraction par équations intégrales*, ENSTA (3rd year), Master AMS (M2) and Master MS2SC (M2)

Marc Bonnet

- *Problème inverses et Identification*, Master MS2SC (M2)

Anne-Sophie Bonnet-Ben Dhia

- *Fonctions de variable complexe*, ENSTA (1st year)
- *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master AMS (M1)
- *Propagation et diffraction dans les guides d'ondes*, ENSTA (3rd year) and Master AMS (M2)
- *Non Destructive Testing*, Master "Acoustical Engineering" (M2)
- *Propagation des ondes élastiques dans les solides*, Master MS2SC (M2)

Laurent Bourgeois

- *Outils élémentaires pour l'analyse des équations aux dérivées partielles*, ENSTA (1st year)
- *Fonctions de variable complexe*, ENSTA (1st year)
- *Complétion de données et identification dans les problèmes gouvernés par des équations aux dérivées partielles*, ENSTA (3rd year) and Master AMS (M2)

Stéphanie Chaillat

- *Introduction à l'environnement UNIX*, ENSTA (1st year)
- *Systèmes d'exploitation*, ENSTA (1st year)
- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
- *Résolution des problèmes de diffraction par équations intégrales*, ENSTA (3rd year), Master AMS (M2) and Master MS2SC (M2)

- *Equations intégrales et multipôles rapides*, Ecole doctorale MODES (Univ. Paris Est, Marne la Vallée)

Colin Chambeyron

- *Remise à niveau en maths*, Licence (1st year), Paris-Dauphine University
- *Outils mathématiques*, Licence (L1), Paris-Dauphine University
- *Analyse - Optimisation*, Licence (L1), Paris-Dauphine University
- *Algèbre linéaire*, Licence (L2), Paris-Dauphine University

Patrick Ciarlet

- *Analyse et approximation par éléments finis d'EDP*, ENSTA (2nd year) and Master AMS (M1)
- *Préformation filière ModSim*, ENSTA (3rd year)
- *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)

Luiz Faria

- *La méthode des éléments finis*, ENSTA (2nd year) and Master AMS (M1)

Sonia Fliss

- *La méthode des éléments finis*, ENSTA (2nd year) and Master AMS (M1)
- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
- *Homogénéisation périodique*, ENSTA (3rd year), Master AMS(M2), Masters ANEDP, M4S
- *Propagation des ondes dans des milieux périodiques*, ENSTA (3rd year) and Master AMS (M2)

Laure Giovangigli

- *Introduction aux probabilités et aux statistiques*, ENSTA (1st year)
- *Martingales et algorithmes stochastiques*, ENSTA (2nd year)
- *Calcul stochastique*, ENSTA (3rd year) and Master MMMEF (M2)

Christophe Hazard

- *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
- *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master AMS (M1)

Patrick Joly

- *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
- *Fonctions de variable complexe*, ENSTA (1st year)
- *Analyse fonctionnelle*, ENSTA (2nd year) and Master AMS (M2)
- *Propagation des ondes dans des milieux périodiques*, ENSTA (3rd year) and Master AMS (M2)

Maryna Kachanovska

- *Fonctions de variable complexe*, ENSTA (1st year)
- *Analyse fonctionnelle*, ENSTA (2nd year) and Master AMS (M1)
- *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)

Nicolas Kielbasiewicz

- *Programmation scientifique en C++*, ENSTA (2nd year) and Master AMS (M1)

- *Projet de simulation numérique*, ENSTA (2nd year) and Master AMS (M1)
- *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)

Eric Lunéville

- *Introduction au calcul scientifique*, ENSTA (2nd year).
- *Programmation scientifique en C++*, ENSTA (2nd year) and Master AMS (M1)
- *Projet de simulation numérique*, ENSTA (2nd year) and Master AMS (M1)
- *Propagation et diffraction dans les guides d'ondes*, ENSTA (3rd year) and Master AMS (M2)

Jean-François Mercier

- *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
- *Fonctions de variable complexe*, ENSTA (1st year)
- *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master AMS (M1)

Axel Modave

- *Calcul scientifique à haute performance*, ENSTA (2nd year) and Master AMS (M1)
- *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)

### 9.2.2. Supervision

PhD: Yohanes Tjandrawidjaja, "*Quelques contributions à l'analyse de la Half-Space Matching Method pour les problèmes de diffraction et son extension aux plaques 3D élastiques*", December 2019, A.-S. Bonnet-Ben Dhia, S. Fliss and V. Baronian

PhD in progress: Sandrine Paolantoni, "*Analyse spectrale et simulation numérique de la diffraction électromagnétique par des métamatériaux*", October 2016, C. Hazard

PhD in progress: Emile Parolin, "*Non overlapping domain decomposition methods with non local transmission conditions for electromagnetic wave propagation*", October 2017, P. Joly and X. Claeys

PhD in progress: Clément Bénéteau, "*Asymptotic analysis of time harmonic Maxwell equations in presence of metamaterials*", October 2017, S. Fliss and X. Claeys

PhD in progress: Hajer Methenni, "*Mathematical modelling and numerical method for the simulation of ultrasound structural health monitoring of composite plates*", October 2017, S. Fliss and S. Impériale

PhD in progress: Damien Mavaleix-Marchessoux, "*Modeling of the fluid-structure interaction resulting from a remote underwater explosion*", December 2017, M. Bonnet and S. Chaillat

PhD in progress: Nicolas Trafny, "*Development of semi-analytical models to predict the noise produced by turbulence-edges interactions*", April 2018, J.-F. Mercier and B. Cotté

PhD in progress: Damien Chicaud, "*Méthodes de décomposition de domaine pour la résolution de problèmes harmoniques d'ondes électromagnétiques en milieux complexes*", October 2018, P. Ciarlet and A. Modave

PhD in progress: Mahran Rihani, "*Équations de Maxwell en présence de méta-matériaux*", November 2018, A.-S. Bonnet-Ben Dhia and L. Chesnel

PhD in progress: Akram Beni Hamad, "*Propagation d'ondes électromagnétiques dans les câbles coaxiaux*", Septembre 2019, S. Imperiale, P. Joly and M. Khenissi

PhD in progress: Jean-François Fritsch, "*Imagerie dans les guides d'ondes enfouis*", Octobre 2019, L. Bourgeois and C. Hazard

PhD in progress: Amandine Boucart "*Modélisation d'une couche mince de nanoparticules réparties aléatoirement pour les ondes électromagnétiques*", Octobre 2019, S. Fliss and L. Giovangigli

## 9.3. Popularization

### 9.3.1. Internal or external Inria responsibilities

- A.-S. Bonnet-Ben Dhia is a member of the *bureau du comité des équipes-projets* (BCEP)
- P. Joly is a member of the *commission consultative paritaire scientifique* of Inria
- M. Kachanovska is a member of the *comité scientifique* of Inria-Saclay

### 9.3.2. Education

- Permanent members of POEMS are involved in the management of the engineering program at ENSTA Paris, the program in applied mathematics at IP Paris, the master 2 program "*Analyse, Modélisation et Simulation*" (M1 AMS) and the master 2 program "*Modélisation et Simulation en Mécanique des Structures et Systèmes Couplés*" (M2 MS2SC) of Université Paris-Saclay:
  - M. Bonnet: co-chair of the M2 MS2SC;
  - L. Bourgeois: co-chair 1A ENSTA until August 2019; co-responsible 2A ENSTA since Septembre 2019; co-chair of the M1 in applied mathematics since Septembre 2019;
  - P. Ciarlet: co-chair 3A ENSTA; deputy head of the M2 AMS until August 2019; coordinator of the master program in applied mathematics at IP Paris;
  - S. Fliss: co-chair of the M2 AMS since septembre 2019.

### 9.3.3. Interventions

- P. Joly presented a talk entitled "*Mathématiques autour de la musique et d'un piano*" during a scientific session at the *Musée des arts et métiers* (March 12, 2019 at Paris) and at the IECL (October 17, 2019 at Nancy).

## 10. Bibliography

### Publications of the year

#### Doctoral Dissertations and Habilitation Theses

- [1] S. CHAILLAT. *Contributions to the modelling of acoustic and elastic wave propagation in large-scale domains with boundary element methods*, ENS Paris Saclay, March 2019, Habilitation à diriger des recherches, <https://hal.archives-ouvertes.fr/tel-02090861>
- [2] S. FLISS. *Wave propagation in periodic media : mathematical analysis and numerical simulation*, Université Paris Sud (Paris 11), January 2019, Habilitation à diriger des recherches, <https://hal.archives-ouvertes.fr/tel-02394976>

#### Articles in International Peer-Reviewed Journals

- [3] F. AMLANI, S. CHAILLAT, A. LOSEILLE. *An efficient preconditioner for adaptive Fast Multipole accelerated Boundary Element Methods to model time-harmonic 3D wave propagation*, in "Computer Methods in Applied Mechanics and Engineering", 2019, vol. 352, n<sup>o</sup> 1, pp. 189-210 [DOI : 10.1016/j.cma.2019.04.026], <https://hal.archives-ouvertes.fr/hal-02113702>
- [4] W. AQUINO, M. BONNET. *Analysis of the error in constitutive equation approach for time-harmonic elasticity imaging*, in "SIAM Journal on Applied Mathematics", 2019, vol. 79, pp. 822-849 [DOI : 10.1137/18M1231237], <https://hal.archives-ouvertes.fr/hal-01948668>



- [5] D. H. BAFFET, M. J. GROTE, S. IMPERIALE, M. KACHANOVSKA. *Energy Decay and Stability of a Perfectly Matched Layer For the Wave Equation*, in "Journal of Scientific Computing", November 2019 [DOI : 10.1007/s10915-019-01089-9], <https://hal.archives-ouvertes.fr/hal-01865484>
- [6] M. BONNET, F. CAKONI. *Analysis of topological derivative as a tool for qualitative identification*, in "Inverse Problems", 2019, vol. 35, n<sup>o</sup> 104007, <https://arxiv.org/abs/1811.10538> [DOI : 10.1088/1361-6420/AB0B67], <https://hal.archives-ouvertes.fr/hal-01929050>
- [7] M. BONNET, E. DEMALDENT. *The eddy current model as a low-frequency, high-conductivity asymptotic form of the Maxwell transmission problem*, in "Computers and Mathematics with Applications", 2019, vol. 77, n<sup>o</sup> 8, pp. 2145-2161 [DOI : 10.1016/J.CAMWA.2018.12.006], <https://hal.archives-ouvertes.fr/hal-01956433>
- [8] J. P. BORTHAGARAY, P. CIARLET. *On the convergence in  $H^1$ -norm for the fractional Laplacian*, in "SIAM Journal on Numerical Analysis", 2019, vol. 57, pp. 1723-1743, <https://hal.inria.fr/hal-01912092>
- [9] L. BOURGEOIS, L. CHESNEL. *On quasi-reversibility solutions to the Cauchy problem for the Laplace equation: regularity and error estimates*, in "ESAIM: Mathematical Modelling and Numerical Analysis", 2019, forthcoming [DOI : 10.1051/M2AN/2019073], <https://hal.inria.fr/hal-02385487>
- [10] L. BOURGEOIS, L. CHESNEL, S. FLISS. *On well-posedness of time-harmonic problems in an unbounded strip for a thin plate model*, in "Communications in Mathematical Sciences", December 2019, <https://hal.archives-ouvertes.fr/hal-01883560>
- [11] L. BOURGEOIS, D. PONOMAREV, J. DARDÉ. *An inverse obstacle problem for the wave equation in a finite time domain*, in "Inverse Problems and Imaging", April 2019, vol. 19, n<sup>o</sup> 2, pp. 377-400 [DOI : 10.3934/IPI.2019019], <https://hal.archives-ouvertes.fr/hal-01818956>
- [12] A. DANSOU, S. MOUHOUBI, C. CHAZALLON, M. BONNET. *Modeling multicrack propagation by the fast multipole symmetric Galerkin BEM*, in "Engineering Analysis with Boundary Elements", September 2019, vol. 106, pp. 309-319 [DOI : 10.1016/J.ENGANABOUND.2019.05.019], <https://hal.archives-ouvertes.fr/hal-02145098>
- [13] B. DELOURME, S. FLISS, P. JOLY, E. VASILEVSKAYA. *Trapped modes in thin and infinite ladder like domains. Part 2 : asymptotic analysis and numerical application*, in "Asymptotic Analysis", 2019, forthcoming, <https://hal.archives-ouvertes.fr/hal-01822437>
- [14] L. FRÉROT, M. BONNET, J.-F. MOLINARI, G. ANCIAUX. *A Fourier-accelerated volume integral method for elastoplastic contact*, in "Computer Methods in Applied Mechanics and Engineering", 2019, vol. 351, pp. 951-976, <https://arxiv.org/abs/1811.11558> [DOI : 10.1016/J.CMA.2019.04.006], <https://hal.archives-ouvertes.fr/hal-01963034>
- [15] I. LABARCA, L. FARIA, C. PÉREZ-ARANCIBIA. *Convolution quadrature methods for time-domain scattering from unbounded penetrable interfaces*, in "Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences", July 2019, vol. 475, n<sup>o</sup> 2227, 20190029 p. [DOI : 10.1098/RSPA.2019.0029], <https://hal.archives-ouvertes.fr/hal-02429472>
- [16] A. MAUREL, J.-F. MERCIER, T. K. PHAM, J.-J. MARIGO, A. OURIR. *Enhanced resonance of sparse arrays of Helmholtz resonators—Application to perfect absorption*, in "Journal of the Acoustical Society of America", April 2019, vol. 145, n<sup>o</sup> 4, pp. 2552-2560 [DOI : 10.1121/1.5098948], <https://hal.inria.fr/hal-02381074>

- [17] A. MODAVE, C. GEUZAINÉ, X. ANTOINE. *Corner treatments for high-order local absorbing boundary conditions in high-frequency acoustic scattering*, in "Journal of Computational Physics", 2020, vol. 401, 109029 p. [DOI : 10.1016/J.JCP.2019.109029], <https://hal.archives-ouvertes.fr/hal-01925160>
- [18] F. MONTEGHETTI, G. HAINE, D. MATIGNON. *Asymptotic stability of the multidimensional wave equation coupled with classes of positive-real impedance boundary conditions*, in "Mathematical Control and Related Fields", December 2019, vol. 9, n<sup>o</sup> 4, pp. 759-791 [DOI : 10.3934/MCRF.2019049], <https://hal.archives-ouvertes.fr/hal-02362852>
- [19] C. PÉREZ-ARANCIBIA, L. FARIA, C. TURC. *Harmonic density interpolation methods for high-order evaluation of Laplace layer potentials in 2D and 3D*, in "Journal of Computational Physics", January 2019, vol. 376, pp. 411-434 [DOI : 10.1016/J.JCP.2018.10.002], <https://hal.archives-ouvertes.fr/hal-02429461>
- [20] C. PÉREZ-ARANCIBIA, C. TURC, L. FARIA. *Planewave Density Interpolation Methods for 3D Helmholtz Boundary Integral Equations*, in "SIAM Journal on Scientific Computing", July 2019, vol. 41, n<sup>o</sup> 4, pp. A2088-A2116 [DOI : 10.1137/19M1239866], <https://hal.archives-ouvertes.fr/hal-02429685>

### Invited Conferences

- [21] M. CASSIER, P. JOLY, M. KACHANOVSKA. *Mathematical models for dispersive electromagnetic waves*, in "19w5209 - Herglotz-Nevanlinna Theory Applied to Passive, Causal and Active Systems", Banff International Research Station, Banff, Canada, October 2019, <https://hal.archives-ouvertes.fr/hal-02401033>

### Conferences without Proceedings

- [22] G. BECK, M. BONNAUD, J. BENOIT. *Recovering underlying graph for networks of 1D waveguides by reflectometry and transferometry*, in "14th International Conference on Mathematical and Numerical Aspects of Wave Propagation", Vienna, Austria, 2019, <https://hal.archives-ouvertes.fr/hal-02414861>
- [23] G. BECK, S. IMPERIALE, P. JOLY. *Effective models for non-perfectly conducting thin coaxial cables*, in "Waves 2019 - 14th International Conference on Mathematical and Numerical Aspects of Wave Propagation", Vienna, Austria, 2019, <https://hal.archives-ouvertes.fr/hal-02414849>
- [24] A. MODAVE, X. ANTOINE, C. GEUZAINÉ. *An efficient domain decomposition method with cross-point treatment for Helmholtz problems*, in "CSMA 2019 - 14e Colloque National en Calcul des Structures", Giens (Var), France, May 2019, <https://hal.archives-ouvertes.fr/hal-02353415>
- [25] A. MODAVE, X. ANTOINE, A. ROYER, C. GEUZAINÉ. *An efficient domain decomposition method with cross-point treatment for Helmholtz problems*, in "WAVES 2019 - 14th International Conference on Mathematical and Numerical Aspects of Wave Propagation", Vienna, Austria, August 2019, <https://hal.archives-ouvertes.fr/hal-02353312>

### Scientific Books (or Scientific Book chapters)

- [26] A.-S. BONNET-BEN DHIA, S. FLISS, Y. TJANDRAWIDJAJA. *Numerical analysis of the Half-Space Matching method with Robin traces on a convex polygonal scatterer*, in "Maxwell's equations: Analysis and numerics", Radon Series on Computational and Applied Mathematics, De Gruyter, May 2019, vol. 24, <https://hal.inria.fr/hal-01793511>

## Other Publications

- [27] J. ALBELLA MARTÍNEZ, S. IMPERIALE, P. JOLY, J. RODRÍGUEZ. *Numerical Analysis of a Method for Solving 2D Linear Isotropic Elastodynamics with Free Boundary Condition using Potentials and Finite Elements*, November 2019, working paper or preprint [DOI : 10.1007/s10915-018-0768-9], <https://hal.inria.fr/hal-02345808>
- [28] L. BOURGEOIS, C. HAZARD. *On well-posedness of scattering problems in a Kirchhoff-Love infinite plate*, October 2019, working paper or preprint, <https://hal-ensta-paris.archives-ouvertes.fr/hal-02334004>
- [29] L. BOURGEOIS, A. RECOQUILLAY. *The Linear Sampling Method for Kirchhoff-Love Infinite Plates*, August 2019, working paper or preprint, <https://hal.inria.fr/hal-02269910>
- [30] R. BUNOIU, L. CHESNEL, K. RAMDANI, M. RIHANI. *Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients*, December 2019, working paper or preprint, <https://hal.inria.fr/hal-02421312>
- [31] P. CIARLET. *On the approximation of electromagnetic fields by edge finite elements. Part 3: sensitivity to coefficients*, July 2019, working paper or preprint, <https://hal.inria.fr/hal-02276430>
- [32] P. JOLY, M. KACHANOVSKA. *Transparent boundary conditions for wave propagation in fractal trees: convolution quadrature approach (extended report)*, August 2019, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-02265345>
- [33] F. D. KPADONOU, S. CHAILLAT, P. CIARLET. *On the efficiency of nested GMRES preconditioners for 3D acoustic and elastodynamic H-matrix accelerated Boundary Element Methods*, December 2019, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-02415902>
- [34] A. MODAVE, A. ROYER, X. ANTOINE, C. GEUZAINÉ. *An optimized Schwarz domain decomposition method with cross-point treatment for time-harmonic acoustic scattering*, January 2020, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-02432422>
- [35] A. NICOLOPOULOS, M. CAMPOS PINTO, B. DESPRÉS, P. CIARLET. *Degenerate elliptic equations for resonant wave problems*, July 2019, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-02142631>
- [36] C. PÉREZ-ARANCIBIA, C. TURC, L. FARIA, C. SIDERIS. *Planewave Density Interpolation Methods for the EFIE on Simple and Composite Surfaces*, October 2019, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-02429487>