

Inria

Activity Report 2019

Project-Team GEOSTAT

Geometry and Statistics in acquisition data

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
**Optimization, machine learning and
statistical methods**

Table of contents

1. Team, Visitors, External Collaborators	1
2. Overall Objectives	2
3. Research Program	3
3.1. General methodology	3
3.2. Turbulence in interstellar clouds and Earth observation data	4
3.3. Causal modeling	7
3.4. Speech analysis	7
3.5. Excitable systems: analysis of physiological time series	7
3.6. Data-based identification of characteristic scales and automated modeling	8
4. Application Domains	9
5. Highlights of the Year	11
6. New Software and Platforms	11
6.1. Flux	11
6.2. FluidExponents	11
6.3. classifemo	12
6.4. superres	12
6.5. EdgeReconstruct	12
6.6. ProximalDenoising	13
6.7. Amuencha	13
6.8. Manzana	13
7. New Results	14
7.1. Excitable systems	14
7.2. Differential diagnosis between atypical Parkinsonian syndromes	14
7.3. Turbulent dynamics of ocean upwelling	14
7.4. Fourier approach to Lagrangian vortex detection	15
7.5. Surface mixing and biological activity	15
7.6. Contribution and influence of coherent mesoscale eddies off the North-West African upwelling on the open ocean	15
7.7. Defining coherent vortices from particles trajectories	16
7.8. Soft hydrogel particle packings	16
7.9. Feature based reconstruction model for fluorescence microscopy image denoising	16
7.10. InnovationLab with I2S, sparse signals & optimisation	17
8. Bilateral Contracts and Grants with Industry	17
9. Partnerships and Cooperations	17
9.1. Regional Initiatives	17
9.2. National Initiatives	17
9.3. European Initiatives	17
9.4. International Initiatives	18
9.4.1. Inria exploratory action	18
9.4.2. Participation in Other International Programs	18
9.5. Introduction	18
9.5.1. Visits of International Scientists	18
9.5.2. Visits to International Teams	18
10. Dissemination	19
10.1. Introduction	19
10.1.1. Member of the Editorial Boards	19
10.1.2. Invited Talks	19
10.2. Introduction	19
10.2.1. Teaching	19

10.2.2. Supervision	19
11. Bibliography	19

Project-Team GEOSTAT

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2. Overall Objectives

2.1. Overall Objectives

GEOSTAT is a research project which investigates the analysis of some classes of natural complex signals (physiological time series, turbulent universe and earth observation data sets) by determining, in acquired signals, the properties that are predicted by commonly admitted or new physical models best fitting the phenomenon. Consequently, when statistical properties discovered in the signals do not match closely enough those predicted by accepted physical models, we question the validity of existing models or propose, whenever possible, modifications or extensions of existing models. We will give, in the sequel, a detailed example of this approach and methodology.

An important aspect of this methodological approach is that we don't rely on a predetermined "universal" signal processing model to analyze natural complex signals. Instead, we take into consideration existing approaches in nonlinear signal processing (wavelets, multifractal analysis tools such as log-cumulants or micro-canonical multifractal formalism, time frequency analysis etc.) which are used to determine the micro structures or other micro features inside the acquired signals. Then, statistical analysis of these micro data are determined and compared to expected behaviour from theoretical physical models used to describe the phenomenon from which the data is acquired. From there different possibilities can be contemplated:

- The statistics match behaviour predicted by the model: complexity parameters predicted by the model are extracted from signals to analyze the dynamics of underlying phenomena. Examples: analysis of turbulent data sets in Oceanography and Astronomy.
- The signals displays statistics that cannot be attainable by the common lore of accepted models: how to extend or modify the models according to the behaviour of observed signals ? Example: electrical activity of heart signal analysis (see infra).

GEOSTAT is a research project in nonlinear signal processing which develops on these considerations: it considers the signals as the realizations of complex extended dynamical systems. The driving approach is to describe the relations between complexity (or information content) and the geometric organization of information in a signal. For instance, for signals which are acquisitions of turbulent fluids, the organization of information may be related to the effective presence of a multiscale hierarchy of coherent structures, of multifractal nature, which is strongly related to intermittency and multiplicative cascade phenomena ; the determination of this geometric organization unlocks key nonlinear parameters and features associated to these signals; it helps understand their dynamical properties and their analysis. We use this approach to

derive novel solution methods for super-resolution and data fusion in Universe Sciences acquisitions [12]. Another example can be found in signal analysis of the electrical activity of the heart, where we find the distribution of activation points in a signal during episodes of atrial fibrillation (with strengthening from feature selection and Bayesian learning see below). Specific advances are obtained in GEOSTAT in using this type of statistical/geometric approach to get validated dynamical information of signals acquired in Universe Sciences, e.g. Oceanography or Astronomy. The research in GEOSTAT encompasses nonlinear signal processing and the study of emergence in complex systems, with a strong emphasis on geometric approaches to complexity. Consequently, research in GEOSTAT is oriented towards the determination, in real signals, of quantities or phenomena, usually unattainable through linear methods, that are known to play an important role both in the evolution of dynamical systems whose acquisitions are the signals under study, and in the compact representations of the signals themselves.

2.1.1.

Signals studied in GEOSTAT belong to two broad classes:

- Acquisitions in Astronomy and Earth Observation.
- Physiological time series.

3. Research Program

3.1. General methodology

Fully Developed Turbulence (FDT) Turbulence at very high Reynolds numbers; systems in FDT are beyond deterministic chaos, and symmetries are restored in a statistical sense only, and multi-scale correlated structures are landmarks. Generalizing to more random uncorrelated multi-scale structured turbulent fields.

Compact Representation Reduced representation of a complex signal (dimensionality reduction) from which the whole signal can be reconstructed. The reduced representation can correspond to points randomly chosen, such as in Compressive Sensing, or to geometric localization related to statistical information content (framework of reconstructible systems).

Sparse representation The representation of a signal as a linear combination of elements taken in a dictionary (frame or Hilbertian basis), with the aim of finding as less as possible non-zero coefficients for a large class of signals.

Universality class In theoretical physics, the observation of the coincidence of the critical exponents (behaviour near a second order phase transition) in different phenomena and systems is called universality. Universality is explained by the theory of the renormalization group, allowing for the determination of the changes followed by structured fluctuations under rescaling, a physical system is the stage of. The notion is applicable with caution and some differences to generalized out-of-equilibrium or disordered systems. Non-universal exponents (without definite classes) exist in some universal slowing dynamical phenomena like the glass transition and kindred. As a consequence, different macroscopic phenomena displaying multiscale structures (and their acquisition in the form of complex signals) may be grouped into different sets of generalized classes.

Every signal conveys, as a measure experiment, information on the physical system whose signal is an acquisition of. As a consequence, it seems natural that signal analysis or compression should make use of physical modelling of phenomena: the goal is to find new methodologies in signal processing that goes beyond the simple problem of interpretation. Physics of disordered systems, and specifically physics of (spin) glasses is putting forward new algorithmic resolution methods in various domains such as optimization, compressive sensing etc. with significant success notably for NP hard problem heuristics. Similarly, physics of turbulence introduces phenomenological approaches involving multifractality. Energy cascades are indeed closely related

to geometrical manifolds defined through random processes. At these structures' scales, information in the process is lost by dissipation (close to the lower bound of inertial range). However, all the cascade is encoded in the geometric manifolds, through long or short distance correlations depending on cases. How do these geometrical manifold structures organize in space and time, in other words, how does the scale entropy cascades itself ? To unify these two notions, a description in term of free energy of a generic physical model is sometimes possible, such as an elastic interface model in a random nonlinear energy landscape : This is for instance the correspondence between compressible stochastic Burgers equation and directed polymers in a disordered medium. Thus, trying to unlock the fingerprints of cascade-like structures in acquired natural signals becomes a fundamental problem, from both theoretical and applicative viewpoints.

To illustrate the general methodology undertaken, let us focus on an example conducted in the study of physiological time series: the analysis of signals recorded from the electrical activity of the heart in the general setting of Atrial Fibrillation (AF). AF is a cardiac arrhythmia characterized by rapid and irregular atrial electrical activity with a high clinical impact on stroke incidence. Best available therapeutic strategies combine pharmacological and surgical means. But when successful, they do not always prevent long-term relapses. Initial success becomes all the more tricky to achieve as the arrhythmia maintains itself and the pathology evolves into sustained or chronic AF. This raises the open crucial issue of deciphering the mechanisms that govern the onset of AF as well as its perpetuation. We have developed a wavelet-based multi-scale strategy to analyze the electrical activity of human hearts recorded by catheter electrodes, positioned in the coronary sinus (CS), during episodes of chronic AF. We have computed the so-called multifractal spectra using two variants of the wavelet transform modulus maxima method, the moment (partition function) method and the magnitude cumulant method (checking confidence intervals with surrogate data). Application of these methods to long time series recorded in a patient with chronic AF provides quantitative evidence of the multifractal intermittent nature of the electric energy of passing cardiac impulses at low frequencies, *i.e.* for times ($> \sim 0.5$ s) longer than the mean interbeat ($\simeq 10^{-1}$ s). We have also reported the results of a two-point magnitude correlation analysis which infers the absence of a multiplicative time-scale structure underlying multifractal scaling. The electric energy dynamics looks like a "multifractal white noise" with quadratic (log-normal) multifractal spectra. *These observations challenge concepts of functional reentrant circuits in mechanistic theories of AF.* A transition is observed in the computed multifractal spectra which group according to two distinct areas, consistently with the anatomical substrate binding to the CS, namely the left atrial posterior wall, and the ligament of Marshall which is innervated by the ANS. These negative results challenge also the existing models, which by principle cannot explain such results. As a consequence, we go beyond the existing models and propose a mathematical model of a denervated heart where the kinetics of gap junction conductance alone induces a desynchronization of the myocardial excitable cells, accounting for the multifractal spectra found experimentally in the left atrial posterior wall area (devoid of ANS influence).

3.2. Turbulence in interstellar clouds and Earth observation data

The research described in this section is a collaboration effort of GEOSTAT, CNRS LEGOS (Toulouse), CNRS LAM (Marseille Laboratory for Astrophysics), MERCATOR (Toulouse), IIT Roorkee, Moroccan Royal Center for Teledetection (CRST), Moroccan Center for Science CNRST, Rabat University, University of Heidelberg. Researchers involved:

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The analysis and modeling of natural phenomena, specially those observed in geophysical sciences and in astronomy, are influenced by statistical and multiscale phenomenological descriptions of turbulence; indeed these descriptions are able to explain the partition of energy within a certain range of scales. A particularly important aspect of the statistical theory of turbulence lies in the discovery that the support of the energy transfer is spatially highly non uniform, in other terms it is *intermittent* [70]. Because of the absence of localization of the Fourier transform, linear methods are not successful to unlock the multiscale structures and cascading properties of variables which are of primary importance as stated by the physics of the phenomena. This is the reason why new approaches, such as DFA (Detrended Fluctuation Analysis), Time-frequency analysis, variations on curvelets [66] etc. have appeared during the last decades. Recent advances in dimensionality reduction, and notably in Compressive Sensing, go beyond the Nyquist rate in sampling theory using nonlinear reconstruction, but data reduction occur at random places, independently of geometric localization of information content, which can be very useful for acquisition purposes, but of lower impact in signal analysis. We are successfully making use of a microcanonical formulation of the multifractal theory, based on predictability and reconstruction, to study the turbulent nature of interstellar molecular or atomic clouds. Another important result obtained in GEOSTAT is the effective use of multiresolution analysis associated to optimal inference along the scales of a complex system. The multiresolution analysis is performed on dimensionless quantities given by the *singularity exponents* which encode properly the geometrical structures associated to multiscale organization. This is applied successfully in the derivation of high resolution ocean dynamics, or the high resolution mapping of gaseous exchanges between the ocean and the atmosphere; the latter is of primary importance for a quantitative evaluation of global warming. Understanding the dynamics of complex systems is recognized as a new discipline, which makes use of theoretical and methodological foundations coming from nonlinear physics, the study of dynamical systems and many aspects of computer science. One of the challenges is related to the question of *emergence* in complex systems: large-scale effects measurable macroscopically from a system made of huge numbers of interactive agents [31], [61]. Some quantities related to nonlinearity, such as Lyapunov exponents, Kolmogorov-Sinai entropy etc. can be computed at least in the phase space [32]. Consequently, knowledge from acquisitions of complex systems (which include *complex signals*) could be obtained from information about the phase space. A result from F. Takens [67] about strange attractors in turbulence has motivated the theoretical determination of nonlinear characteristics associated to complex acquisitions. Emergence phenomena can also be traced inside complex signals themselves, by trying to localize information content geometrically. Fundamentally, in the nonlinear analysis of complex signals there are broadly two approaches: characterization by attractors (embedding and bifurcation) and time-frequency, multiscale/multiresolution approaches. In real situations, the phase space associated to the acquisition of a complex phenomenon is unknown. It is however possible to relate, inside the signal's domain, local predictability to local reconstruction [13] and to deduce relevant information associated to multiscale geophysical signals [14]. A multiscale organization is a fundamental feature of a complex system, it can be for example related to the cascading properties in turbulent systems. We make use of this kind of description when analyzing turbulent signals: intermittency is observed within the inertial range and is related to the fact that, in the case of FDT (fully developed turbulence), symmetry is restored only in a statistical sense, a fact that has consequences on the quality of any nonlinear signal representation by frames or dictionaries.

The example of FDT as a standard "template" for developing general methods that apply to a vast class of complex systems and signals is of fundamental interest because, in FDT, the existence of a multiscale hierarchy \mathcal{F}_h which is of multifractal nature and geometrically localized can be derived from physical considerations. This geometric hierarchy of sets is responsible for the shape of the computed singularity spectra, which in turn is related to the statistical organization of information content in a signal. It explains scale invariance, a characteristic feature of complex signals. The analogy from statistical physics comes from the fact that singularity exponents are direct generalizations of *critical exponents* which explain the macroscopic properties of a system around critical points, and the quantitative characterization of *universality classes*, which allow the definition of methods and algorithms that apply to general complex signals and systems, and not only turbulent signals: signals which belong to a same universality class share common statistical organization. During the past decades, canonical approaches permitted the development of a well-established analogy taken from thermodynamics in the analysis of complex signals: if \mathcal{F} is the free energy, \mathcal{T} the temperature measured

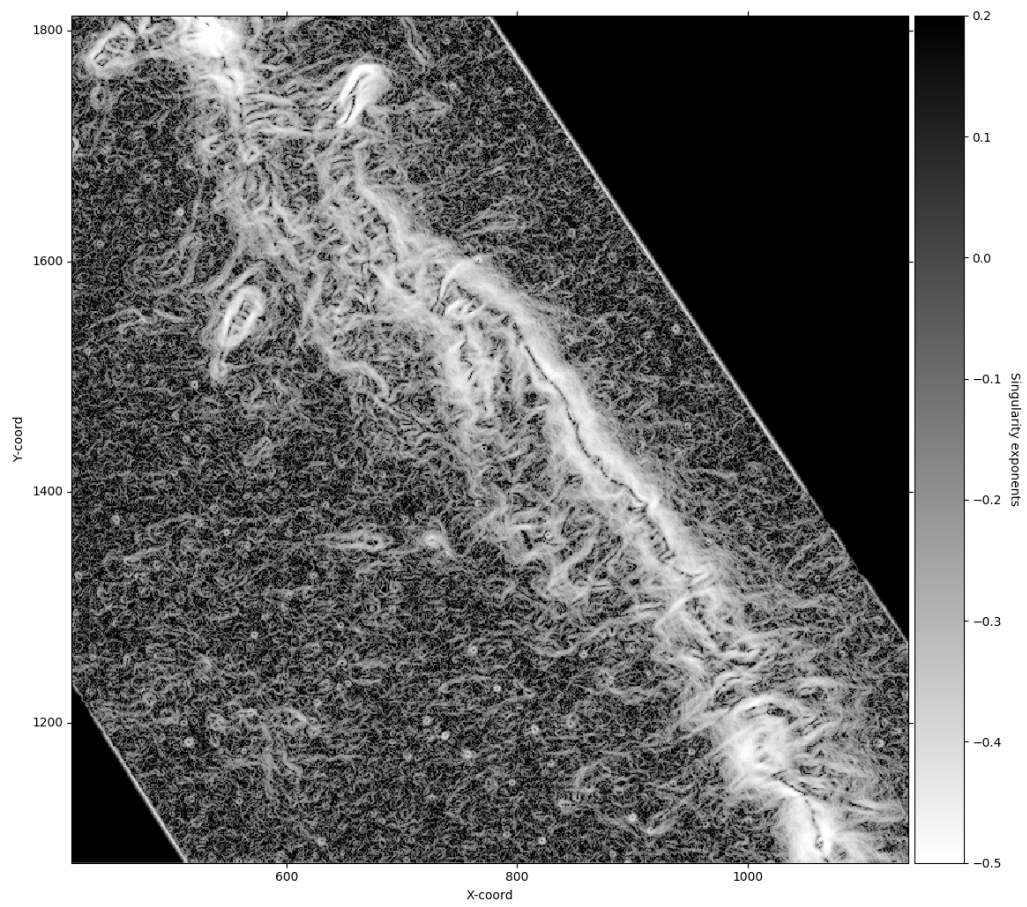


Figure 1. Visualization of the singularity exponents, computed on a edge-aware filtered *Musca Herschel* observation map .

in energy units, \mathcal{U} the internal energy per volume unit \mathcal{S} the entropy and $\hat{\beta} = 1/\mathcal{T}$, then the scaling exponents associated to moments of intensive variables $p \rightarrow \tau_p$ corresponds to $\hat{\beta}\mathcal{F}$, $\mathcal{U}(\hat{\beta})$ corresponds to the singularity exponents values, and $\mathcal{S}(\mathcal{U})$ to the singularity spectrum [27]. The research goal is to be able to determine universality classes associated to acquired signals, independently of microscopic properties in the phase space of various complex systems, and beyond the particular case of turbulent data [53].

We show in figure 1 the result of the computation of singularity exponents on an *Herschel* astronomical observation map (the Musca galactic cloud) which has been edge-aware filtered using sparse L^1 filtering to eliminate the cosmic infrared background (or CIB), a type of noise that can modify the singularity spectrum of a signal.

3.3. Causal modeling

The team is working on a new class of models for modeling physical systems, starting from measured data and accounting for their dynamics [40]. The idea is to statistically describe the evolution of a system in terms of causally-equivalent states; states that lead to the same predictions [33]. Transitions between these states can be reconstructed from data, leading to a theoretically-optimal predictive model [63]. In practice, however, no algorithm is currently able to reconstruct these models from data in a reasonable time and without substantial discrete approximations. Recent progress now allows a continuous formulation of predictive causal models. Within this framework, more efficient algorithms may be found. The broadened class of predictive models promises a new perspective on structural complexity in many applications.

3.4. Speech analysis

Phonetic and sub-phonetic analysis: We developed a novel algorithm for automatic detection of Glottal Closure Instants (GCI) from speech signals using the Microcanonical Multiscale Formalism (MMF). This state of the art algorithm is considered as a reference in this field. We made a Matlab code implementing it available to the community ([link](#)). Our approach is based on the Microcanonical Multiscale Formalism. We showed that in the case of clean speech, our algorithm performs almost as well as a recent state-of-the-art method. In presence of different types of noises, we showed that our method is considerably more accurate (particularly for very low SNRs). Moreover, our method has lower computational times does not rely on an estimate of pitch period nor any critical choice of parameters. Using the same MMF, we also developed a method for phonetic segmentation of speech signal. We showed that this method outperforms state of the art ones in term of accuracy and efficiency.

Pathological speech analysis and classification: we made a critical analysis of some widely used methodologies in pathological speech classification. We then introduced some novel methods for extracting some common features used in pathological speech analysis and proposed more robust techniques for classification.

Speech analysis of patients with Parkinsonism: with our collaborators from the Czech Republic, we started preliminary studies of some machine learning issues in the field essentially due the small amount of training data.

3.5. Excitable systems: analysis of physiological time series

The research described in this section is a collaboration effort of GEOSTAT, CNRS LOMA (Laboratoire Ondes et Matière d'Aquitaine) and Laboratory of Physical Foundation of Strength, Institute of Continuous Media Mechanics (Perm, Russia Federation).

AF is an arrhythmia originating in the rapid and irregular electrical activity of the atria (the heart's two upper chambers) that causes their pump function to fail, increasing up to fivefold the risk of embolic stroke. The prevailing electrophysiological concepts describing tachy-arrhythmias are more than a century old. They involve abnormal automaticity and conduction [52]. Initiation and maintenance are thought to arise from a vulnerable substrate prone to the emergence of multiple self-perpetuating reentry circuits, also called "multiple wavelets" [59], [60]. Reentries may be driven structurally, for instance because of locally high fibrous tissue

content which badly conducts, or functionally because of high spatial dispersion of decreased refractoriness and APD [58]. The latter is coined the leading circle concept with the clinically more relevant notion of a critical “wavelength” (in fact the length) of the cardiac impulse [26], [65], [62], [30]. The related concept of vulnerability was originally introduced to uncover a physiological substrate evolving from normality to pathology. It was found in vulnerable patients that high rate frequency would invariably lead to functional disorder as cardiac cells would no longer properly adapt their refractoriness [29]. Mathematical models have managed to exhibit likewise phenomena, with the generation of breaking spiral waves in various conditions [49], [54]. The triggering role of abnormal ectopic activity of the pulmonary veins has been demonstrated on patients with paroxysmal AF resistant to drug therapy [48], but its origin still remains poorly understood. This region is highly innervated with sympathetic and parasympathetic stimulation from the ANS [68], [69], [28]. In particular, Coumel et al. [39], [38] have revealed the pathophysiological role of the vagal tone on a vulnerable substrate. It is frequently observed that rapid tachycardia of ectopic origin transits to AF. This is known to result from electrical remodeling. As described for the first time by Allessie et al. [25], remodeling is a transient and reversible process by which the impulse properties such as its refractory period are altered during the course of the arrhythmia, promoting its perpetuation: “AF begets AF” [71]. Under substantial beating rate increase, cells may undergo remodeling to overcome the toxicity of their excessive intercellular calcium loading, by a rapid down regulation (a few minutes) of their L-type calcium membrane current. Moreover, other ionic channel functions are also modified such as the potassium channel function, inducing a change in the conduction properties including the conduction velocity. The intercellular coupling at the gap junction level shows also alterations of their connexin expression and dispersion.

Wavelet-based methods (WTMM, log-cumulants, two point scale correlations), and confidence statistical methodology, have been applied to catheter recordings in the coronary sinus vein right next to the left atria of a small sample of patients with various conditions, and exhibit clear multifractal scaling without cross-scale correlation, which are coined “multifractal white noise”, and that can be grouped according to two anatomical regions. One of our main result was to show that this is incompatible with the common lore for atrial fibrillation based on so-called circuit reentries. We used two declinations of a wavelet-based multiscale method, the moment (partition function) method and the magnitude cumulant method, as originally introduced in the field of fully developed turbulence. In the context of cardiac physiology, this methodology was shown to be valuable in assessing congestive heart failure from the monitoring of sinus heart rate variability [50]. We develop a model such that the substrate function is modulated by the kinetics of conduction. A simple reversible mechanism of short term remodeling under rapid pacing is demonstrated, by which ionic overload acts locally (dynamical feedback) on the kinetics of gap junction conductance. The whole process may propagate and pervade the myocardium via electronic currents, becoming desynchronized. In a new description, we propose that circuit reentries may well exist before the onset of fibrillation, favoring onset but not contributing directly to the onset and perpetuation. By contrast, cell-to-cell coupling is considered fundamentally dynamical. The rationale stems from the observation that multifractal scaling necessitates a high number of degrees of freedom (tending to infinity with system size), which can originate in excitable systems in hyperbolic spatial coupling.

3.6. Data-based identification of characteristic scales and automated modeling

Data are often acquired at the highest possible resolution, but that scale is not necessarily the best for modeling and understanding the system from which data was measured. The intrinsic properties of natural processes do not depend on the arbitrary scale at which data is acquired; yet, usual analysis techniques operate at the acquisition resolution. When several processes interact at different scales, the identification of their characteristic scales from empirical data becomes a necessary condition for properly modeling the system. A classical method for identifying characteristic scales is to look at the work done by the physical processes, the energy they dissipate over time. The assumption is that this work matches the most important action of each process on the studied natural system, which is usually a reasonable assumption. In the framework of time-frequency analysis [45], the power of the signal can be easily computed in each frequency band, itself matching a temporal scale.

However, in open and dissipating systems, energy dissipation is a prerequisite and thus not necessarily the most useful metric to investigate. In fact, most natural, physical and industrial systems we deal with fall in this category, while balanced quasi-static assumptions are practical approximation only for scales well below the characteristic scale of the involved processes. Open and dissipative systems are not locally constrained by the inevitable rise in entropy, thus allowing the maintaining through time of mesoscopic ordered structures. And, according to information theory [47], more order and less entropy means that these structures have a higher information content than the rest of the system, which usually gives them a high functional role.

We propose to identify characteristic scales not only with energy dissipation, as usual in signal processing analysis, but most importantly with information content. Information theory can be extended to look at which scales are most informative (e.g. multi-scale entropy [37], ϵ -entropy [36]). Complexity measures quantify the presence of structures in the signal (e.g. statistical complexity [42], MPR [56] and others [44]). With these notions, it is already possible to discriminate between random fluctuations and hidden order, such as in chaotic systems [41], [56]. The theory of how information and structures can be defined through scales is not complete yet, but the state of art is promising [43]. Current research in the team focuses on how informative scales can be found using collections of random paths, assumed to capture local structures as they reach out [35].

Building on these notions, it should also possible to fully automate the modeling of a natural system. Once characteristic scales are found, causal relationships can be established empirically. They are then clustered together in internal states of a special kind of Markov models called ϵ -machines [42]. These are known to be the optimal predictors of a system, with the drawback that it is currently quite complicated to build them properly, except for small system [64]. Recent extensions with advanced clustering techniques [34], [46], coupled with the physics of the studied system (e.g. fluid dynamics), have proved that ϵ -machines are applicable to large systems, such as global wind patterns in the atmosphere [51]. Current research in the team focuses on the use of reproducing kernels, coupled possibly with sparse operators, in order to design better algorithms for ϵ -machines reconstruction. In order to help with this long-term project, a collaboration is ongoing with J. Crutchfield lab at UC Davis.

4. Application Domains

4.1. Sparse signals & optimization

This research topic involves **Geostat** team and is used to set up an InnovationLab with **I2S company**

Sparsity can be used in many ways and there exist various sparse models in the literature; for instance minimizing the l_0 quasi-norm is known to be an NP-hard problem as one needs to try all the possible combinations of the signal's elements. The l_1 norm, which is the convex relation of the l_0 quasi-norm results in a tractable optimization problem. The l_p pseudo-norms with $0 < p < 1$ are particularly interesting as they give closer approximation of l_0 but result in a non-convex minimization problem. Thus, finding a global minimum for this kind of problem is not guaranteed. However, using a non-convex penalty instead of the l_1 norm has been shown to improve significantly various sparsity-based applications. Nonconvexity has a lot of statistical implications in signal and image processing. Indeed, natural images tend to have a heavy-tailed (kurtotic) distribution in certain domains such as wavelets and gradients. Using the l_1 norm comes to consider a Laplacian distribution. More generally, the hyper-Laplacian distribution is related to the l_p pseudo-norm ($0 < p < 1$) where the value of p controls how the distribution is heavy-tailed. As the hyper-Laplacian distribution for $0 < p < 1$ represents better the empirical distribution of the transformed images, it makes sense to use the l_p pseudo-norms instead of l_1 . Other functions that better reflect heavy-tailed distributions of images have been used as well such as Student-t or Gaussian Scale Mixtures. The internal properties of natural images have helped researchers to push the sparsity principle further and develop highly efficient algorithms for restoration, representation and coding. Group sparsity is an extension of the sparsity principle where data is clustered into groups and each group is sparsified differently. More specifically, in many cases, it makes sense to follow a certain structure when sparsifying by forcing similar sets of points to be zeros or non-zeros simultaneously. This is typically true for natural images that represent coherent structures. The

concept of group sparsity has been first used for simultaneously shrinking groups of wavelet coefficients because of the relations between wavelet basis elements. Lastly, there is a strong relationship between sparsity, nonpredictability and scale invariance.

We have shown that the two powerful concepts of sparsity and scale invariance can be exploited to design fast and efficient imaging algorithms. A general framework has been set up for using non-convex sparsity by applying a first-order approximation. When using a proximal solver to estimate a solution of a sparsity-based optimization problem, sparse terms are always separated in subproblems that take the form of a proximal operator. Estimating the proximal operator associated to a non-convex term is thus the key component to use efficient solvers for non-convex sparse optimization. Using this strategy, only the shrinkage operator changes and thus the solver has the same complexity for both the convex and non-convex cases. While few previous works have also proposed to use non-convex sparsity, their choice of the sparse penalty is rather limited to functions like the l_p pseudo-norm for certain values of $p \geq 0.5$ or the Minimax Concave (MC) penalty because they admit an analytical solution. Using a first-order approximation only requires calculating the (super)gradient of the function, which makes it possible to use a wide range of penalties for sparse regularization. This is important in various applications where we need a flexible shrinkage function such as in edge-aware processing. Apart from non-convexity, using a first-order approximation makes it easier to verify the optimality condition of proximal operator-based solvers via fixed-point interpretation. Another problem that arises in various imaging applications but has attracted less works is the problem of multi-sparsity, when the minimization problem includes various sparse terms that can be non-convex. This is typically the case when looking for a sparse solution in a certain domain while rejecting outliers in the data-fitting term. By using one intermediate variable per sparse term, we show that proximal-based solvers can be efficient. We give a detailed study of the Alternating Direction Method of Multipliers (ADMM) solver for multi-sparsity and study its properties. The following subjects are addressed and receive new solutions:

1. **Edge aware smoothing:** given an input image g , one seeks a smooth image u "close" to g by minimizing:

$$\operatorname{argmin}_u \frac{\lambda}{2} \|u - g\|_2^2 + \psi(\nabla u)$$

where ψ is a sparsity-inducing non-convex function and λ a positive parameter. Splitting and alternate minimization lead to the sub-problems:

$$\begin{aligned} \text{(sp1)} & : v^{(k+1)} \leftarrow \operatorname{argmin}_v \psi(v) + \frac{\beta}{2} \|\nabla u^{(k)} - v\|_2^2 \\ \text{(sp2)} & : u^{(k+1)} \leftarrow \operatorname{argmin}_u \lambda \|u - g\|_2^2 + \beta \|\nabla u - v^{(k+1)}\|_2^2. \end{aligned}$$

We solve sub-problem (sp2) through deconvolution and efficient estimation via separable filters and warm-start initialization for fast GPU implementation, and sub-problem (sp1) through non-convex proximal form.

2. **Structure-texture separation:** design of an efficient algorithm using non-convex terms on both the data-fitting and the prior. The resulting problem is solved via a combination of Half-Quadratic (HQ) and Maximization-Minimization (MM) methods. We extract challenging texture layers outperforming existing techniques while maintaining a low computational cost. Using spectral sparsity in the framework of low-rank estimation, we propose to use robust Principal Component Analysis (RPCA) to perform robust separation on multi-channel images such as glare and artifacts removal of flash/no-flash photographs. As in this case, the matrix to decompose has much less columns than lines, we propose to use a QR decomposition trick instead of a direct singular value decomposition (SVD) which makes the decomposition faster.

3. **Robust integration:** in many applications, we need to reconstruct an image from corrupted gradient fields. The corruption can take the form of outliers only when the vector field is the result of transformed gradient fields (low-level vision), or mixed outliers and noise when the field is estimated from corrupted measurements (surface reconstruction, gradient camera, Magnetic Resonance Imaging (MRI) compressed sensing, etc.). We use non-convexity and multi-sparsity to build efficient integrability enforcement algorithms. We present two algorithms : 1) a local algorithm that uses sparsity in the gradient field as a prior together with a sparse data-fitting term, 2) a non-local algorithm that uses sparsity in the spectral domain of non-local patches as a prior together with a sparse data-fitting term. Both methods make use of a multi-sparse version of the Half-Quadratic solver. The proposed methods were the first in the literature to propose a sparse regularization to improve integration. Results produced with these methods significantly outperform previous works that use no regularization or simple l_1 minimization. Exact or near-exact recovery of surfaces is possible with the proposed methods from highly corrupted gradient fields with outliers.
4. **Learning image denoising:** deep convolutional networks that consist in extracting features by repeated convolutions with high-pass filters and pooling/downsampling operators have shown to give near-human recognition rates. Training the filters of a multi-layer network is costly and requires powerful machines. However, visualizing the first layers of the filters shows that they resemble wavelet filters, leading to sparse representations in each layer. We propose to use the concept of scale invariance of multifractals to extract invariant features on each sparse representation. We build a bi-Lipschitz invariant descriptor based on the distribution of the singularities of the sparsified images in each layer. Combining the descriptors of each layer in one feature vector leads to a compact representation of a texture image that is invariant to various transformations. Using this descriptor that is efficient to calculate with learning techniques such as classifiers combination and artificially adding training data, we build a powerful texture recognition system that outperforms previous works on 3 challenging datasets. In fact, this system leads to quite close recognition rates compared to latest advanced deep nets while not requiring any filters training.

5. Highlights of the Year

5.1. Highlights of the Year

- Inria's exploratory action "TRACME" led by N. Brodu, starting October 2019.

6. New Software and Platforms

6.1. Fluex

KEYWORDS: Signal - Signal processing

SCIENTIFIC DESCRIPTION: Fluex is a package consisting of the Microcanonical Multiscale Formalism for 1D, 2D 3D and 3D+t general signals.

FUNCTIONAL DESCRIPTION: Fluex is a C++ library developed under Gforge. Fluex is a library in nonlinear signal processing. Fluex is able to analyze turbulent and natural complex signals, Fluex is able to determine low level features in these signals that cannot be determined using standard linear techniques.

- Participants: Hussein Yahia and Rémi Paties
- Contact: Hussein Yahia
- URL: <http://fluex.gforge.inria.fr/>

6.2. FluidExponents

KEYWORDS: Signal processing - Wavelets - Fractal - Spectral method - Complexity

FUNCTIONAL DESCRIPTION: FluidExponents is a signal processing software dedicated to the analysis of complex signals displaying multiscale properties. It analyzes complex natural signals by use of nonlinear methods. It implements the multifractal formalism and allows various kinds of signal decomposition and reconstruction. One key aspect of the software lies in its ability to evaluate key concepts such as the degree of unpredictability around a point in a signal, and provides different kinds of applications. The software can be used for times series or multidimensional signals.

- Participants: Antonio Turiel and Hussein Yahia
- Contact: Hussein Yahia
- URL: <svn+ssh://fluidexponents@scm.gforge.inria.fr/svn/fluidexponents/FluidExponents>

6.3. classifemo

KEYWORDS: Classification - Audio

FUNCTIONAL DESCRIPTION: Classifies vocal audio signals. Classifemo extracts characteristics from vocal audio signals. These characteristics are extracted from signals of different type: initially these were emotion databases, but it can also process signals recorded from patients with motor speech disorders. The software can train usual classifiers (SVM, random forests, etc) on these databases as well as classify new signals.

- Participants: Khalid Daoudi and Nicolas Brodu
- Contact: Khalid Daoudi
- URL: <http://algo.inria.fr/app/emotionclassifierprototype>

6.4. superres

Super-Resolution of multi-spectral and multi-resolution images

KEYWORD: Multiscale

SCIENTIFIC DESCRIPTION: This resolution enhancement method is designed for multispectral and multiresolution images, such as these provided by the Sentinel-2 satellites (but not only). Starting from the highest resolution bands, band-dependent information (reflectance) is separated from information that is common to all bands (geometry of scene elements). This model is then applied to unmix low-resolution bands, preserving their reflectance, while propagating band-independent information to preserve the sub-pixel details.

FUNCTIONAL DESCRIPTION: This super-resolution software for multi-spectral images consists of: - A core C++ library, which can be used directly - A Python module interface to this library - A Java JNI interface to the library - An end-user Python script for super-resolving Sentinel-2 images - An end-user plugin for the widely used SNAP software of the ESA.

- Participant: Nicolas Brodu
- Contact: Nicolas Brodu
- URL: <http://nicolas.brodu.net/recherche/superres/index.html>

6.5. EdgeReconstruct

Edge Reconstruction With UPM Manifold

KEYWORDS: 2D - Fractal - Signal processing

FUNCTIONAL DESCRIPTION: EdgeReconstruct is a software that reconstructs a complex signal from the computation of most unpredictable points in the framework of the Microcanonical Multifractal Formalism. The quality of the reconstruction is also evaluated. The software is a companion of a paper published in 2013: <https://hal.inria.fr/hal-00924137>.

- Contact: Suman Kumar Maji
- URL: <https://geostat.bordeaux.inria.fr/index.php/downloads.html>

6.6. ProximalDenoising

KEYWORDS: 2D - Image filter - Filtering - Minimizing overall energy - Noise - Signal processing - Image reconstruction - Image processing

SCIENTIFIC DESCRIPTION: Image filtering is contemplated in the form of a sparse minimization problem in a non-convex setting. Given an input image I , one seeks to compute a denoised output image u such that u is close to I in the L_2 norm. To do so, a minimization term is added which favors sparse gradients for output image u . Imposing sparse gradients lead to a non-convex minimization term: for instance a pseudo-norm L_p with $0 < p < 1$ or a Cauchy or Welsh function. Half-quadratic algorithm is used by adding a new variable in the minimization functional which leads to two sub-problems, the first sub-problem is non-convex and solved by use of proximal operators. The second sub-problem can be written in variational form, and is best solved in Fourier space: it takes the form of a deconvolution operator whose kernel can be approximated by a finite sum of separable filters. This solution method produces excellent computation times even on big images.

FUNCTIONAL DESCRIPTION: Use of proximal and non quadratic minimization. GPU implementation.

RELEASE FUNCTIONAL DESCRIPTION: This software implements H. Badri PhD thesis results.

- Authors: Marie Martin, Chiheb Sakka, Hussein Yahia, Nicolas Brodu, Gabriel Augusto Zebadua Garcia and Khalid Daoudi
- Partner: Innovative Imaging Solutions I2S
- Contact: Hussein Yahia
- URL: https://gitlab.inria.fr/marmarti/i2s_geostat_C

6.7. Amuencha

Musical analyzer and singing training

KEYWORDS: Audio - Real-time rendering

SCIENTIFIC DESCRIPTION: The typical audio analyzer uses Fast Fourier Transforms (FFT) in order to find the frequency content. The problem with this approach is that notes follow the logarithm of the frequencies. ... while the FFT is linear in frequency. This results in a loss of precision. Even worse, the window function used for localizing the frequencies in time is often non-optimal, which increases this precision loss. Generally, nearby frequencies « bleed on » the one being analyzed. Amuencha does not use FFT. I first create a bank of filters, each centered on one frequency to analyze. These filters are complex exponentials convoluted with a Kaiser window, which support is set according to the frequency to analyze. Once these filters are applied to the signal, I use a time-frequency reassignment technique in order to exploit the complex phase of the signal. This method combines the information from nearby filters, but with a different phase, in order to restore very precisely the frequency content with the smallest possible delay.

FUNCTIONAL DESCRIPTION: Amuencha gathers notes separated by an octave along the same directions of a spiral, so that chords clearly stand out, even reversed. You can also record yourself with a microphone (in red) while playing some recording (in blue), so you can work your tuning :-)

RELEASE FUNCTIONAL DESCRIPTION: Initial version

- Author: Nicolas Brodu
- Contact: Nicolas Brodu
- URL: <https://nicolas.brodu.net/programmation/amuencha/>

6.8. Manzana

KEYWORDS: 2D - Image processing - Filtering

SCIENTIFIC DESCRIPTION: Software library developed in the framework of I2S-GEOSTAT innovationlab and made of high-level image processing functionalities based on sparsity and non-convex optimization.

FUNCTIONAL DESCRIPTION: Library of software in image processing: filtering, hdr, inpainting etc.

- Partner: Innovative Imaging Solutions I2S
- Contact: Hussein Yahia

7. New Results

7.1. Excitable systems

Participants: G. Attuel , E. Gerasimova-Chechkina , F. Argoul , H. Yahia , A. Arnéodo .

In a companion paper (I. Multifractal analysis of clinical data), we used a wavelet-based multiscale analysis to reveal and quantify the multifractal intermittent nature of the cardiac impulse energy in the low frequency range 2Hz during atrial fibrillation (AF). It demarcated two distinct areas within the coronary sinus (CS) with regionally stable multifractal spectra likely corresponding to different anatomical substrates. The electrical activity also showed no sign of the kind of temporal correlations typical of cascading processes across scales, thereby indicating that the multifractal scaling is carried by variations in the large amplitude oscillations of the recorded bipolar electric potential. In the present study, to account for these observations, we explore the role of the kinetics of gap junction channels (GJCs), in dynamically creating a new kind of imbalance between depolarizing and repolarizing currents. We propose a one-dimensional (1D) spatial model of a denervated myocardium, where the coupling of cardiac cells fails to synchronize the network of cardiac cells because of abnormal transjunctional capacitive charging of GJCs. We show that this non-ohmic nonlinear conduction 1D modeling accounts quantitatively well for the "multifractal random noise" dynamics of the electrical activity experimentally recorded in the left atrial posterior wall area. We further demonstrate that the multifractal properties of the numerical impulse energy are robust to changes in the model parameters.

Publications: *Frontiers in Physiology*, *Frontiers*, 2019, 10, pp.480 (1-18), [HAL](#)

Second international Summer Institute on Network Physiology, Jul 2019 [HAL](#)

7.2. Differential diagnosis between atypical Parkinsonian syndromes

Participants: B. Das , K. Daoudi , J. Kemplir , J. Rusz .

In the early stage of disease, the symptoms of Parkinson's disease (PD) are similar to atypical Parkinsonian syndromes (APS). The early differential diagnosis between PD and APS and within APS is thus a very challenging task. It turns out that speech disorder is an early and common symptom to PD and APS. The goal of research is to develop a digital marker based on speech analysis in order to assist the neurologists in their diagnosis. We addressed the problem of differential diagnosis between two APS, Progressive Supranuclear Palsy (PSP) and Multiple System Atrophy (MSA). Using linear discriminant analysis we designed an hypokinetic and an ataxic dysarthria measure which yield a discrimination between PSP and MSA with a high accuracy (88%). This result indicates that hypokinetic and ataxic dysarthria convey valuable discriminative information when mutually considered.

Publication: ICASSP 2019 [HAL](#)

7.3. Turbulent dynamics of ocean upwelling

Participants: A. El Aouni , V. Garçon , J. Sudre , H. Yahia , K. Daoudi , K. Minaoui .

The region along the NorthWest African coast (20° N to 36° N and 4° W to 19° W) is characterized by a persistent and variable upwelling phenomenon almost all year round. In this article, the upwelling features are investigated using an algorithm dedicated to delimit the upwelling area from thermal and biological satellite observations. This method has been developed specifically for sea-surface temperature (SST) images, since they present a high latitudinal variation, which is not present in chlorophyll-a concentration images. Developing on the proposed approach, the spatial and temporal variations of the main physical and biological upwelling patterns are studied. Moreover, a study on the upwelling dynamics, which explores the interplay between the upwelling spatiotemporal extents and intensity, is presented, based on a 14-year time archive of weekly SST and chlorophyll-a concentration data.

Publication: IEEE Transactions on Geoscience and Remote Sensing, [HAL](#)

7.4. Fourier approach to Lagrangian vortex detection

Participants: A. El Aouni , H. Yahia , K. Daoudi , K. Minaoui .

We study the transport properties of coherent vortices over a finite time duration. Here we reveal that such vortices can be identified based on frequency-domain representation of Lagrangian trajectories. We use Fourier analysis to convert particles' trajectories from their time domain to a presentation in the frequency domain. We then identify and extract coherent vortices as material surfaces along which particles' trajectories share similar frequencies. Our method identifies all coherent vortices in an automatic manner, showing high vortices' monitoring capacity. We illustrate our new method by identifying and extracting Lagrangian coherent vortices in different two-and three-dimensional flows.

Publication: Chaos, American Institute of Physics, [HAL](#)

7.5. Surface mixing and biological activity

Participants: A. El Aouni , K. Daoudi , H. Yahia , K. Minaoui , A. Benazzouz .

Near-shore water along the NorthWest African margin is one of the world's major upwelling regions. It is associated with physical structures of oceanic fronts which influence the biological productivity. The study of these coherent structures in connection with chlorophyll concentration data is of fundamental importance for understanding the spatial distributions of the plankton. In this work, we study the horizontal stirring and mixing in different upwelling areas using Lagrangian coherent structures (LCSs). These LCSs are calculated using the recent geodesic theory of LCSs. We use these LCSs to study the link between the chlorophyll fronts concentrations and surface mixing, based on 10 years of satellite data. These LCSs move with the flow as material lines, thus the horizontal mixing is calculated from the intersection of these LCSs with the finite time Lyapunov exponents (FTLEs) maps. We compare our results with those of a recent study conducted over the same area, but based on Finite Size Lyapunov Exponents (FSLEs) whose output is a plot of scalar distributions. We discuss the differences between FSLE and geodesic theory of LCS. The latter yields analytical solutions of LCSs, while FSLEs can only provide LCSs for sharp enough ridges of nearly constant height.

Publication: Chaos, American Institute of Physics, [HAL](#)

7.6. Contribution and influence of coherent mesoscale eddies off the North-West African upwelling on the open ocean

Participants: A. El Aouni , K. Daoudi , H. Yahia , K. Minaoui .

Eastern Boundary Upwelling zones include some of the most productive ecosystems in the world, particularly the NorthWest (NW) African upwelling which presents one of the world's major upwelling regions. This latter is forced by the equator-ward trade winds which are known to exhibit mesoscale instabilities; thus, in addition to upwelled cold and nutrient-rich deep waters, significant energy is transferred into mesoscale fronts and eddies in the upper ocean. Oceanic structures of type eddies are well known to stir and mix surrounding water masses. However, they can also carry and transport organic matter and marine in a coherent manner. Here, we are interested in those that remain coherent. The Aim of this work, is to understand the impact and the contribution of the mesoscale eddies off the NW African margin on the open ocean. Our approach to analyze such coherent eddies is based upon the use of our recently developed technique from nonlinear dynamics theory, which is capable of identifying coherent vortices and their centers in an automatic manner. The role of these mesoscale eddies is investigated based on a statistical study of eddies properties off NW African margin (cyclone/anticyclone, their lifetimes, traveled distance, translational speeds, quantity of water masses transported to the open ocean...). This statistical study is carried out over a sets of 24 years (spans from January 1993 to December 2016) of sea surface velocity field derived from satellite surface altimetry under the geostrophic approximation.

Publication: SIAM Conference on Mathematics of Planet Earth, (MPE18) [HAL](#)

7.7. Defining coherent vortices from particles trajectories

Participants: A. El Aouni , H. Yahia , K. Daoudi , K. Minaoui .

Tracer patterns in the ocean, such as sea surface temperature, chlorophyll concentration and salinity suggest the emergence of coherence even in the ocean, typically with fluxes dominated by advective transport over diffusion. Mesoscale eddies are known to govern advective transport in the ocean, with typical horizontal scales $\mathcal{O}(100km)$ and timescales of $\mathcal{O}(weeks)$. These oceanic structures are omnipresent in the ocean and usually exhibit different properties to their surroundings. They are known to stir and mix surroundings water masses as well as by their ability to trap and carry fluid properties in a coherent manner. As the effect of these mesoscale eddies on the global circulation is remarkable, we focused on studying and understanding the dynamic transport properties of these coherent oceanic structures. For this reason, we have developed a Lagrangian method to identify and extract Lagrangian coherent vortices. The method analyzes particles' trajectories to identify vortices' boundaries and their attractor centers. Indeed, it is based on a decomposition of particle trajectory into two parts: closed curves which give information about uniformly rotating flow, and one that describes the mean displacement. The former part yields an objective measure of material rotation. We define Lagrangian coherent vortex as closed material lines in which fluid parcels complete the same polar rotation. This turns out to be filled with outward-increasing closed contours of the Lagrangian Averaged Closed Curve Length.

Publications: 3 articles submitted.

7.8. Soft hydrogel particle packings

Participants: J. Bares , N. Brodu , H. Zheng , J. A. Dijksman .

We provide the raw data from several years worth of effort on index matching experiments on soft hydrogel particle packings exposed to various loading conditions. Many of the data sets have not before been used, described explicitly in scientific publications or even analyzed. We will present a general overview of methods used to perform the imaging experiments. We provide particle-level information extracted from the 3D images, including contact forces and particle shapes, along with complementary boundary force measurements. We also provide a final working version of the code used to extract these microscopic features.

Publication: *Releasing data from mechanical tests on three dimensional hydrogel sphere packings*, Granular Matter, [HAL](#)

7.9. Feature based reconstruction model for fluorescence microscopy image denoising

Participants: S. K. Maji , H. Yahia .

: the advent of Fluorescence Microscopy over the last few years have dramatically improved the problem of visualization and tracking of specific cellular objects for biological inference. But like any other imaging system, fluorescence microscopy has its own limitations. The resultant images suffer from the effect of noise due to both signal dependent and signal independent factors, thereby limiting the possibility of biological inferencing. Denoising is a class of image processing algorithms that aim to remove noise from acquired images and has gained wide attention in the field of fluorescence microscopy image restoration. In this paper, we propose an image denoising algorithm based on the concept of feature extraction through multifractal decomposition and then estimate a noise free image from the gradients restricted to these features. Experimental results over simulated and real fluorescence microscopy data prove the merit of the proposed approach, both visually and quantitatively.

Publication: Scientific Reports, Nature Publishing Group, [HAL](#)

7.10. InnovationLab with I2S, sparse signals & optimisation

Participants: M. Martin, H. Yahia, A. Zebadua, S. Sakka, N. Brodu, K. Daoudi, A. Cherif [I2S], J. L. Vallancogne [I2S], A. Cailly [I2S], A. Billy [I2S], B. Lebouill [I2S].

During 2019:

Linear inverse problems in image processing, denoising, deconvolution, non-convex optimization, plug, and play algorithms.

HDR (High Dynamic Range) : C++ implementation with OpenCV library; code deposited on gitlab, sharing with I2S company, enhancement of the code architecture. GPU implementation.

Reflecting improvement: development and C++ implementation.

3D reconstruction: implementation and test with openMVG and openMVS libraries. Depth map enhancement through non-convex optimization.

Image smoothing implementation [57].

CPU and GPU patchmatch implementation [55].

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

InnovationLab with I2S company, starting scheduled after 1st 2019 COPIL in January 2019.

9. Partnerships and Cooperations

9.1. Regional Initiatives

Geostat is a member of the GPR ("Grand Projet de Recherche") **ORIGINS** ("Origine, évolution, matière primordiale, nucléosynthèse, complexification, étoiles, planètes, Terre, habitabilité, climat, biodiversité, homininsés, big data, sociologie, médiation scientifique) carried by Laboratoire d'Astrophysique de Bordeaux (LAB) (M. Gargaud). Geostat is involved in the axis "Data Science pour les Sciences de Origins".

9.2. National Initiatives

- ANR project *Voice4PD-MSA*, led by K. Daoudi, which targets the differential diagnosis between Parkinson's disease and Multiple System Atrophy. The total amount of the grant is 468555 euros, from which GeoStat has 203078 euros. The duration of the project is 42 months. Partners: CHU Bordeaux (Bordeaux), CHU Toulouse, IRIT, IMT (Toulouse).
- Prolongation for A. El Aouni in 2019 (4 months) through the program "BOOSTE TON DOC" of the Toubkal PHC project PHC-Toubkal project "Caractérisation multi-capteurs et suivi spatio-temporel de l'Upwelling sur la côte atlantique marocaine par imagerie satellitaire", which finished December 2018.
- GEOSTAT is a member of ISIS (Information, Image & Vision), AMF (Multifractal Analysis) GDRs.
- GEOSTAT is participating in the CNRS IMECO project *Intermittence multi-échelles de champs océaniques : analyse comparative d'images satellitaires et de sorties de modèles numériques*. CNRS call AO INSU 2018. PI: F. Schmitt, DR CNRS, UMR LOG 8187. Duration: 2 years.

9.3. European Initiatives

9.3.1. Collaborations in European Programs, Except FP7 & H2020

GENESIS Program: supported by Deutsche Forschungsgemeinde (DFG) and the Agence national de recherche (ANR). *GENERation and Evolution of Structures in the ISm*. Duration: start 1.5. 2017, 3 years. Coordinator: N. Schneider (I. Physik, Cologne). Other partners: Cologne (R. Simon, N. Schneider, V. Ossenkopf, M. Roellig), LAB (S. Bontemps, A. Roy, L. Bonne, F. Herpin, J. Braine, N. Brouillet, T. Jacq), ATN Canberra (Australia), LERMA Paris (France), MPIfR Bonn (Germany), CEA Saclay (France), ITA/ZAH Heidelberg (Germany), Institute of Astronomy, Cardiff (UK), ESO (Germany, Chile), CfA Harvard (USA), IPAG Grenoble (France), Argelander Institut Bonn (Germany), CASS San Diego (USA), University of Sofia (Bulgaria). Web site: [link](#).

9.4. International Initiatives

9.4.1. Inria exploratory action

TRACME This project focuses on modelling a physical system from measurements on that system. How, starting from observations, to build a reliable model of the system dynamics? When multiple processes interact at different scales, how to obtain a significant model at each of these scales? The goal is to provide a model simple enough to bring some understanding of the system studied, but also a model elaborated enough to allow precise predictions. In order to do so, this project proposes to identify causally equivalent classes of system states, then model their evolution with a stochastic process. Renormalizing these equations is necessary in order to relate the scale of the continuum to that, arbitrary, at which data are acquired. Applications primarily concern natural sciences. PI: N. Brodu.

9.4.2. Participation in Other International Programs

9.4.2.1. IFCAM: Generalization for land cover identification. Geostat and the Indo-French Centre For Applied Mathematics

Land cover classification from satellite imagery is an important application for agriculture, environmental monitoring, tracking changes for emergency, etc. The typical methodology is to train a machine learning algorithm to recognize specified classes (urban, forest, fields, etc...) over regions of interest and classify new images when they become available. This proposal investigates how to use local context and how to best sample the data in order to provide the best generalization ability. Data will be sampled on reference locations and used for training and validation.

PIs: N. Brodu (Geostat) and D. Singh (IIT Roorkee).

Duration: 3 years. Starting 2018.

9.5. Introduction

9.5.1. Visits of International Scientists

- D. Singh [IIT Roorkee, June 2019]

9.5.1.1. Internships

- D. Nash, level L3, intern in June 2019. Supervisor: N. Brodu.

9.5.2. Visits to International Teams

- PhD student A. Rashidi met with Dr Francis Bach of Inria Paris on optimization methods. first meeting was on November 2019.
- A. Rashidi registered for "Inversion et imagerie haute resolution" lectures of Dr. Francois Giovannelli, starting from January 2020.
- A. Rashidi participated in PRAIRE artificial intelligence summer school in October 2019 at Paris.

10. Dissemination

10.1. Introduction

10.1.1. Member of the Editorial Boards

- H. Yahia is Review Editor for the journal *Frontiers in Physiology (Fractal Physiology)*.

10.1.2. Invited Talks

- Journée des Systèmes et de la Matière Complexe 4ème édition, Paris-Saclay, October 2019: G. Attuel gave the presentation *Voir au coeur d'une turbulence forte sans cascade et mourir...*, [link](#).

10.2. Introduction

10.2.1. Teaching

Master : K. Daoudi, Data mining, 20 hours, M2 MIAGE, University of Lorraine.

Master : A. Rashidi, Introduction to deep learning, 12 hours, M1, course given in the team during D. Singh's visit.

10.2.2. Supervision

PhD : A. El Aouni, Lagrangian coherent structures and physical processes of coastal upwelling, Bordeaux University, September 24, 2019, supervisors: H. Yahia, K. Minaoui.

Master : H. Belmajdoub, Upwelling dynamics along the Atlantic coast of Morocco, Rabat University, supervised by A. El Aouni.

PhD in progress : H. Belmajdoub, Upwelling dynamics along the Atlantic coast of Morocco, Rabat University, cosupervised by A. El Aouni.

PhD in progress : Z. El Abidi, caractérisation multicapteur de l'upwelling marocain, Rabat University, cosupervised by A. El Aouni.

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