Activity Report 2019

Project-Team AROMATH

AlgebRe gèOmétrie Modelisation et AlgoriTHmes

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Project-Team AROMATH

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B9.5.1. - Computer science
B9.5.2. - Mathematics

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2. Overall Objectives

2.1. Overall Objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of "real" phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to "act" on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, ... are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects "at home" from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimisation loops where the numerical simulation results are used to improve the "performance" of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world...), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

3. Research Program

3.1. High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in CAGD, to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at knots. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

3.2. Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally, as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods, which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One relies on linearization of algebraic problems based on "elimination of variables" or projection into a space of smaller dimension. Resultant theory provides strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progresses have been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute "only" the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

4. Application Domains

4.1. Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing ...Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axes machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

4.2. Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree ≤ 2).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are

investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

5. New Results

5.1. Truncated Normal Forms for Solving Polynomial Systems: Generalized and Efficient Algorithms

Participant: Bernard Mourrain.

In [16], we consider the problem of finding the isolated common roots of a set of polynomial functions defining a zero-dimensional ideal I in a ring R of polynomials over $\mathbb{C}$. Normal form algorithms provide an algebraic approach to solve this problem. The framework presented in Telen et al. (2018) uses truncated normal forms (TNFs) to compute the algebra structure of R/I and the solutions of I. This framework allows for the use of much more general bases than the standard monomials for R/I. This is exploited in this paper to introduce the use of two special (non-monomial) types of basis functions with nice properties. This allows, for instance, to adapt the basis functions to the expected location of the roots of I. We also propose algorithms for efficient computation of TNFs and a generalization of the construction of TNFs in the case of non-generic zero-dimensional systems. The potential of the TNF method and usefulness of the new results are exposed by many experiments.

This is a joint work with Simon Telen and Marc Van Barel, Department of Computer Science - K.U.Leuven.

5.2. Implicit representations of high-codimension varieties

Participants: Ioannis Emiris, Clément Laroche, Christos Konaxis.

In [8], we study implicitization, which usually focuses on plane curves and (hyper)surfaces, in other words, varieties of codimension 1. We shift the focus on space curves and, more generally, on varieties of codimension larger than 1, and discuss approaches that are not sensitive to base points. Our first contribution is a direct generalization of an implicitization method based on interpolation matrices for objects of high codimension given parametrically or as point clouds. Our result shows the completeness of this approach which, furthermore, reduces geometric operations and predicates to linear algebra computations. Our second, and main contribution is an implicitization method of parametric space curves and varieties of codimension > 1, which exploits the theory of Chow forms to obtain the equations of conical (hyper)surfaces intersecting precisely at the given object. We design a new, practical, randomized algorithm that always produces correct output but possibly with a non-minimal number of surfaces. For space curves, which is the most common case, our algorithm returns 3 surfaces whose polynomials are of near-optimal degree; moreover, computation reduces to a Sylvester resultant. We illustrate our algorithm through a series of examples and compare our Maple code with other methods implemented in Maple. Our prototype is not faster but yields fewer equations and is more robust than Maple's implicitize. Although not optimized, it is comparable with Gröbner bases and matrix representations derived from syzygies, for degrees up to 6.

5.3. Saturation of Jacobian ideals: Some applications to nearly free curves, line arrangements and rational cuspidal plane curves

Participant: Alexandru Dimca.

In [6] we describe the minimal resolution of the ideal I_f, the saturation of the Jacobian ideal of a nearly free plane curve $(C : f) = 0$. In particular, it follows that this ideal I_f can be generated by at most 4 polynomials. Some applications to rational cuspidal plane curves are given, and a natural related question is raised.

This is a joint work with Gabriel Sticlaru (Ovidius University of Constanta).

5.4. Matrix formulae for Resultants and Discriminants of Bivariate Tensor-product Polynomials

Participants: Laurent Busé, Angelos Mantzaflaris.

The construction of optimal resultant formulae for polynomial systems is one of the main areas of research in computational algebraic geometry. However, most of the constructions are restricted to formulae for unmixed polynomial systems, that is, systems of polynomials which all have the same support. Such a condition is restrictive, since mixed systems of equations arise frequently in many problems. Nevertheless, resultant formulae for mixed polynomial systems is a very challenging problem. In [5], we introduce a square, Koszul-type, matrix, the determinant of which is the resultant of an arbitrary (mixed) bivariate tensor-product polynomial system. The formula generalizes the classical Sylvester matrix of two univariate polynomials, since it expresses a map of degree one, that is, the elements of the corresponding matrix are up to sign the coefficients of the input polynomials. Interestingly, the matrix expresses a primal-dual multiplication map, that is, the tensor product of a univariate multiplication map with a map expressing derivation in a dual space. In addition we prove an impossibility result which states that for tensor-product systems with more than two (affine) variables there are no universal degree-one formulae, unless the system is unmixed. Last but not least, we present applications of the new construction in the efficient computation of discriminants and mixed discriminants.

This is joint work with Elias Tsigaridas (Ouragan, Inria).

5.5. Implicitizing rational curves by the method of moving quadrics

Participants: Laurent Busé, Clément Laroche, Fatmanur Yildirim.

In [4], a new technique for finding implicit matrix-based representations of rational curves in arbitrary dimension is introduced. It relies on the use of moving quadrics following curve parameterizations, providing a high-order extension of the implicit matrix representations built from their linear counterparts, the moving planes. The matrices we obtain offer new, more compact, implicit representations of rational curves. Their entries are filled by linear and quadratic forms in the space variables and their ranks drop exactly on the curve. Typically, for a general rational curve of degree d we obtain a matrix whose size is half of the size of the corresponding matrix obtained with the moving planes method. We illustrate the advantages of these new matrices with some examples, including the computation of the singularities of a rational curve.

5.6. Separation bounds for polynomial systems

Participants: Ioannis Emiris, Bernard Mourrain.

In [9] we rely on aggregate separation bounds for univariate polynomials to introduce novel worst-case separation bounds for the isolated roots of zero-dimensional, positive-dimensional, and overdetermined polynomial systems. We exploit the structure of the given system, as well as bounds on the height of the sparse (or toric) resultant, by means of mixed volume, thus establishing adaptive bounds. Our bounds improve upon Canny's Gap theorem [9]. Moreover, they exploit sparseness and they apply without any assumptions on the input polynomial system. To evaluate the quality of the bounds, we present polynomial systems whose root separation is asymptotically not far from our bounds. We apply our bounds to three problems. First, we use them to estimate the bit-size of the eigenvalues and eigenvectors of an integer matrix; thus we provide a new proof that the problem has polynomial bit complexity. Second, we bound the value of a positive polynomial over the simplex: we improve by at least one order of magnitude upon all existing bounds. Finally, we asymptotically bound the number of steps of any purely subdivision-based algorithm that isolates all real roots of a polynomial system.

This is a joint work with E. Tsigaridas (Ouragan).

5.7. Sparse polynomial interpolation: sparse recovery, super resolution, or Prony?

Participant: Bernard Mourrain.

In [12], we show that the sparse polynomial interpolation problem reduces to a discrete super-resolution problem on the n-dimensional torus. Therefore the semidefinite programming approach initiated by Candès & Fernandez-Granda in the univariate case can be applied. We extend their result to the multivariate case, i.e., we show that exact recovery is guaranteed provided that a geometric spacing condition on the supports holds and the number of evaluations are sufficiently many (but not many). It also turns out that the sparse recovery LP-formulation of $\ell 1$-norm minimization is also guaranteed to provide exact recovery *provided that* the evaluations are made in a certain manner and even though the Restricted Isometry Property for exact recovery is not satisfied. (A naive sparse recovery LP-approach does not offer such a guarantee.) Finally we also describe the algebraic Prony method for sparse interpolation, which also recovers the exact decomposition but from less point evaluations and with no geometric spacing condition. We provide two sets of numerical experiments, one in which the super-resolution technique and Prony's method seem to cope equally well with noise, and another in which the super-resolution technique seems to cope with noise better than Prony's method, at the cost of an extra computational burden (i.e. a semidefinite optimization).

This is a joint work with Cédric Josz and Jean-Bernard Lasserre (Équipe Méthodes et Algorithmes en Commande, LAAS).

5.8. Computing minimal Gorenstein covers

Participant: Bernard Mourrain.

In [7], we analyze and present an effective solution to the minimal Gorenstein cover problem: given a local Artin k–algebra $A = k[[x_1, ..., x_n]]/I$, compute an Artin Gorenstein k–algebra $G = k[[x_1, ..., x_n]]/I$ such that $\ell(G) - \ell(A)$ is minimal. We approach the problem by using Macaulay's inverse systems and a modification of the integration method for inverse systems to compute Gorenstein covers. We propose new characterizations of the minimal Gorenstein cover and present a new algorithm for the effective computation of the variety of all minimal Gorenstein covers of A for low Gorenstein colength. Experimentation illustrates the practical behavior of the method.

This is a joint work with Juan Elias and Roser Homs (Dep. de Matematiques i Informatica, Universitat de Barcelona).

5.9. Symmetry Preserving Interpolation

Participants: Erick David Rodriguez Bazan, Evelyne Hubert.

In [22], we address multivariate interpolation in the presence of symmetry. Interpolation is a prime tool in algebraic computation while symmetry is a qualitative feature that can be more relevant to a mathematical model than the numerical accuracy of the parameters. The article shows how to exactly preserve symmetry in multivariate interpolation while exploiting it to alleviate the computational cost. We revisit minimal degree and least interpolation with symmetry adapted bases, rather than monomial bases.This allows to construct bases of invariant interpolation spaces in blocks, capturing the inherent redundancy in the computations.We show that the so constructed symmetry adapted interpolation bases alleviate the computational cost of any interpolation problem and automatically preserve any equivariance of their interpolation problem might have.

5.10. Skew-Symmetric Tensor Decomposition

Participant: Bernard Mourrain.

In [2], we introduce the "skew apolarity lemma" and we use it to give algorithms for the skew-symmetric rank and the decomposition of tensors in $\wedge^d V_{\mathbb{C}}$ with $d \leq 3$ and $\dim V_{\mathbb{C}} \leq 8$. New algorithms to compute the rank and a minimal decomposition of a tri-tensor are also presented.

This is a joint work with Enrique Arrondo (UCM - Universidad Complutense de Madrid, Spain), Alessandra Bernardi (Department of Mathematics, University of Trento, Italy) Pedro Macias Marques (Departamento de Matemática da Universidade de Évora, Spain).

5.11. On the maximal number of real embeddings of minimally rigid graphs in $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{S}^2$

Participants: Ioannis Emiris, Evangelos Bartzos.

In [3], we study the Rigidity theory studies the properties of graphs that can have rigid embeddings in the d-dimensional Euclidean space, or on a sphere and other manifolds which in addition satisfy certain edge length constraints. One of the major open problems in this field is to determine lower and upper bounds on the number of realizations with respect to a given number of vertices. This problem is closely related to the classification of rigid graphs according to their maximal number of real embeddings. In this paper, we are interested in finding edge lengths that can maximize the number of real embeddings of minimally rigid graphs in the plane, space, and on the sphere. We use algebraic formulations to provide upper bounds. To find values of the parameters that lead to graphs with a large number of real realizations, possibly attaining the (algebraic) upper bounds, we use some standard heuristics and we also develop a new method inspired by coupler curves. We apply this new method to obtain embeddings in $\mathbb{R}^3$. One of its main novelties is that it allows us to sample efficiently from a larger number of parameters by selecting only a subset of them at each iteration. Our results include a full classification of the 7-vertex graphs according to their maximal numbers of real embeddings in the cases of the embeddings in $\mathbb{R}^2$ and $\mathbb{R}^3$, while in the case of $\mathbb{S}^2$ we achieve this classification for all 6-vertex graphs. Additionally, by increasing the number of embeddings of selected graphs, we improve the previously known asymptotic lower bound on the maximum number of realizations.

This is a joint work with E. Tsigaridas (Ouragan), and J. Legersky (JK University, Linz, Austria).

5.12. Voronoï diagram of orthogonal polyhedra in two and three dimensions

Participants: Ioannis Emiris, Christina Katsamaki.

In [20], we study Voronoï diagrams, which are a fundamental geometric data structure for obtaining proximity relations. We consider collections of axis-aligned orthogonal polyhedra in two and three-dimensional space under the max-norm, which is a particularly useful scenario in certain application domains. We construct the exact Voronoï diagram inside an orthogonal polyhedron with holes defined by such polyhedra. Our approach avoids creating full-dimensional elements on the Voronoï diagram and yields a skeletal representation of the input object. We introduce a complete algorithm in 2D and 3D that follows the subdivision paradigm relying on a bounding-volume hierarchy; this is an original approach to the problem. The complexity is adaptive and comparable to that of previous methods. Under a mild assumption it is $O(n/D)$ in 2D or $O(na^2/D^2)$ in 3D, where n is the number of sites, namely edges or facets resp., D is the maximum cell size for the subdivision to stop, and a bounds vertex cardinality per facet. We also provide a numerically stable, open-source implementation in Julia, illustrating the practical nature of our algorithm.

The software was developed during Katsamaki's internship in 2018 at Sophia-Antipolis under the supervision of Bernard Mourrain. The problem has been proposed by our industrial collaborator ANSYS Hellas. The paper is based on Katsamaki's MSc thesis.

5.13. Near-Neighbor Preserving Dimension Reduction for Doubling Subsets of L_1

Participants: Ioannis Emiris, Ioannis Psarros.

In [21], we study randomized dimensionality reduction which has been recognized as one of the fundamental techniques in handling high-dimensional data. Starting with the celebrated Johnson-Lindenstrauss Lemma, such reductions have been studied in depth for the Euclidean (L_2) metric, but much less for the Manhattan (L_1) metric. Our primary motivation is the approximate nearest neighbor problem in L_1. We exploit its reduction to the decision-with-witness version, called approximate near neighbor, which incurs a roughly logarithmic overhead. In 2007, Indyk and Naor, in the context of approximate nearest neighbors, introduced the notion of nearest neighbor-preserving embeddings. These are randomized embeddings between two metric spaces with guaranteed bounded distortion only for the distances between a query point and a point set. Such embeddings are known to exist for both L_2 and L_1 metrics, as well as for doubling subsets of L_2. The case that remained open were doubling subsets of L_1. In this paper, we propose a dimension reduction by means of a near neighbor-preserving embedding for doubling subsets of L_1. Our approach is to represent the pointset with a carefully chosen covering set, then randomly project the latter. We study two types of covering sets: c-approximate r-nets and randomly shifted grids, and we discuss the tradeoff between them in terms of preprocessing time and target dimension. We employ Cauchy variables: certain concentration bounds derived should be of independent interest.

This is joint work with Vassilis Margonis (NKUA), and is based on his MSc thesis.

5.14. On the cross-sectional distribution of portfolio returns

Participants: Ioannis Emiris, Apostolos Chalkis.

The aim of the paper [24] is to study the distribution of portfolio returns across portfolios, and for given asset returns. We focus on the most common type of investment, considering portfolios whose weights are non-negative and sum up to 1. We provide algorithms and formulas from computational geometry and the literature on splines to compute the exact values of the probability density function, and of the cumulative distribution function, at any point. We also provide closed form solutions for the computation of its first four moments, and an algorithm to compute the higher moments. All algorithms and formulas allow also for equal asset returns.

This is a joint work with Ludovic Calès (JRC - European Commission - Joint Research Centre, Ispra).

5.15. Enumerating the morphologies of non-degenerate Darboux cyclides

Participant: Bernard Mourrain.

In [19] we provide an enumeration of all possible morphologies of non-degenerate Darboux cyclides. Based on the fact that every Darboux cyclide in $\mathbb{R}^3$ is the stereographic projection of the intersection surface of a sphere and a quadric in $\mathbb{R}^4$, we transform the enumeration problem of morphologies of Darboux cyclides to the enumeration of the algebraic sequences that characterize the intersection of a sphere and a quadric in $\mathbb{R}^4$.

This is a joint work with Mingyang Zhao, Xiaohong Jia (KLMM - Key Laboratory of Mathematics Mechanization, Beijing, China), Changhe Tu (Shandong University, China), Wenping Wang (Computer Graphics Group, Department of Computer Science, Hong Kong, China).

5.16. Anisotropic convolution surfaces

Participants: Alvaro Fuentes Suarez, Evelyne Hubert.

Convolution surfaces with 1D skeletons have been limited to close-to-circular normal sections. The new formalism and method presented in [10] allows for ellipsoidal normal sections. Anisotropy is prescribed on G^1 skeletal curves, chosen as circular splines, by a rotation angle and the three radii of an ellipsoid at each extremity. This lightweight model creates smooth shapes that previously required tweaking the skeleton or supplementing it with 2D pieces. The scale invariance of our formalism achieves excellent radii control and thus lends itself to approximate a variety of shapes. The construction of a scaffold is extended to skeletons with G^1 branches. It projects onto the convolution surface as a quad mesh with skeleton bound edge-flow.

This is a joint work with Cédric Zanni (MFX Inria NGE).

5.17. A non-iterative method for robustly computing the intersections between a line and a curve or surface

Participant: Laurent Busé.

The need to compute the intersections between a line and a high-order curve or surface arises in a large number of finite element applications. Such intersection problems are easy to formulate but hard to solve robustly. In [18], we introduce a non-iterative method for computing intersections by solving a matrix singular value decomposition (SVD) and an eigenvalue problem. That is, all intersection points and their parametric coordinates are determined in one-shot using only standard linear algebra techniques available in most software libraries. As a result, the introduced technique is far more robust than the widely used Newton-Raphson iteration or its variants. The maximum size of the considered matrices depends on the polynomial degree q of the shape functions and is $2q \times 3q$ for curves and $6q^2 \times 8q^2$ for surfaces. The method has its origin in algebraic geometry and has here been considerably simplified with a view to widely used high-order finite elements. In addition, the method is derived from a purely linear algebra perspective without resorting to algebraic geometry terminology. A complete implementation is available from http://bitbucket.org/nitro-project/.

This is joint work with Xiao Xiao and Fehmi Cirak (Cambridge, UK).

5.18. Cooperative Visual-Inertial Sensor Fusion: the Analytic Solution

Participant: Bernard Mourrain.

In [15], we analyze the visual–inertial sensor fusion problem in the cooperative case of two agents, and proves that this sensor fusion problem is equivalent to a simple polynomial equations system that consists of several linear equations and three polynomial equations of second degree. The analytic solution of this polynomial equations system is easily obtained by using an algebraic method. In other words, this letter provides the analytic solution to the visual–inertial sensor fusion problem in the case of two agents. The power of the analytic solution is twofold. From one side, it allows us to determine the relative state between the agents (i.e., relative position, speed, and orientation) without the need of an initialization. From another side, it provides fundamental insights into all the theoretical aspects of the problem. This letter mainly focuses on the first issue. However, the analytic solution is also exploited to obtain basic structural properties of the problem that characterize the observability of the absolute scale and the relative orientation. Extensive simulations and real experiments show that the solution is successful in terms of precision and robustness.

This is a joint work with Agostino Martinelli and Alexander Oliva (CHROMA, Inria Grenoble).

5.19. Overlapping Multi-Patch Structures in Isogeometric Analysis

Participant: Angelos Mantzaflaris.

In isogeometric analysis (IGA) the domain of interest is usually represented by B-spline or NURBS patches, as they are present in standard CAD models. Complex domains can often be represented as a union of simple overlapping subdomains, parameterized by (tensor-product) spline patches. Numerical simulation on such overlapping multi-patch domains is a serious challenge in IGA. To obtain non-overlapping subdomains one would usually reparameterize the domain or trim some of the patches. Alternatively, one may use methods that can handle overlapping subdomains. In [13] we propose a non-iterative, robust and efficient method defined directly on overlapping multi-patch domains. Consequently, the problem is divided into several sub-problems, which are coupled in an appropriate way. The resulting system can be solved directly in a single step. We compare the proposed method with iterative Schwarz domain decomposition approaches and observe that our method reduces the computational cost significantly, especially when handling subdomains with small overlaps. Summing up, our method significantly simplifies the domain parameterization problem, since we can represent any domain of interest as a union of overlapping patches without the need to introduce trimming curves/surfaces. The performance of the proposed method is demonstrated by several numerical experiments for the Poisson problem and linear elasticity in two and three dimensions.

This is a joint work with S. Kargaran, B. Jüttler, S. Kleiss and T. Takacs. (RICAM - Johann Radon Institute for Computational and Applied Mathematics and Institute of Applied Geometry, Linz, Austria)

5.20. First Order Error Correction for Trimmed Quadrature in Isogeometric Analysis

Participant: Angelos Mantzaflaris.

In [23] we develop a specialized quadrature rule for trimmed domains , where the trimming curve is given implicitly by a real-valued function on the whole domain. We follow an error correction approach: In a first step, we obtain an adaptive subdivision of the domain in such a way that each cell falls in a pre-defined base case. We then extend the classical approach of linear approximation of the trimming curve by adding an error correction term based on a Taylor expansion of the blending between the linearized implicit trimming curve and the original one. This approach leads to an accurate method which improves the convergence of the quadrature error by one order compared to piecewise linear approximation of the trimming curve. It is at the same time efficient, since essentially the computation of one extra one-dimensional integral on each trimmed cell is required. Finally, the method is easy to implement, since it only involves one additional line integral and refrains from any point inversion or optimization operations. The convergence is analyzed theoretically and numerical experiments confirm that the accuracy is improved without compromising the computational complexity.

This is joint work with B. Jüttler and F. Scholz. (Institute of Applied Geometry, Linz, Austria).

5.21. Consistent discretization of higher-order interface models for thin layers and elastic material surfaces, enabled by isogeometric cut-cell methods

Participant: Angelos Mantzaflaris.

Many interface formulations, e.g. based on asymptotic thin interphase models or material surface theories, involve higher-order differential operators and discontinuous solution fields. In [11] we are taking first steps towards a variationally consistent discretization framework that naturally accommodates these two challenges by synergistically combining recent developments in isogeometric analysis and cut-cell finite element methods. Its basis is the mixed variational formulation of the elastic interface problem that provides access to jumps in displacements and stresses for incorporating general interface conditions. Upon discretization with smooth splines, derivatives of arbitrary order can be consistently evaluated, while cut-cell meshes enable discontinuous solutions at potentially complex interfaces. We demonstrate via numerical tests for three specific nontrivial interfaces (two regimes of the Benveniste–Miloh classification of thin layers and the Gurtin–Murdoch material surface model) that our framework is geometrically flexible and provides optimal higher-order accuracy in the bulk and at the interface.

This is joint work with Zhilin Han, Changzheng Cheng, (HFUT - Hefei University of Technology, China), Chien-Ting Wu, S. Stoter, S. Mogilevskaya, and D. Schillinger (Department of Civil, Environmental and Geo-Engineering, University of Minnesota, USA).

5.22. Design of Self-Supporting Surfaces with Isogeometric Analysis

Participant: Angelos Mantzaflaris.

Self-supporting surfaces are widely used in contemporary architecture, but their design remains a challenging problem. This paper aims to provide a heuristic strategy for the design of complex self-supporting surfaces. In our method, presented in [17] non-uniform rational B-spline (NURBS) surfaces are used to describe the smooth geometry of the self-supporting surface. The equilibrium state of the surface is derived with membrane shell theory and Airy stresses within the surfaces are used as tunable variables for the proposed heuristic design strategy. The corresponding self-supporting shapes to the given stress states are calculated by the nonlinear isogeometric analysis (IGA) method. Our validation using analytic catenary surfaces shows that the proposed

method finds the correct self-supporting shape with a convergence rate one order higher than the degree of the applied NURBS basis function. Tests on boundary conditions show that the boundary's influence propagates along the main stress directions in the surface. Various self-supporting masonry structures, including models with complex topology, are constructed using the presented method. Compared with existing methods such as thrust network analysis and dynamic relaxation, the proposed method benefits from the advantages of NURBS-based IGA, featuring smooth geometric description, good adaption to complex shapes and increased efficiency of computation.

This is joint work with Yang Xia, Ping Hu (Dalian University of Technology, China), Bert Jüttler (Institute of Applied Geometry, Linz, Austria), Hao Pan (Microsoft Research Asia, China), Wenping Wang (CSE - Department of Computer Science and Engineering, HKUST, Honk Kong, China).

5.23. Low-rank space-time decoupled isogeometric analysis for parabolic problems with varying coefficients

Participant: Angelos Mantzaflaris.

In [14] we present a space-time isogeometric analysis scheme for the discretization of parabolic evolution equations with diffusion coefficients depending on both time and space variables. The problem is considered in a space-time cylinder in $\mathbb{R}^{d+1}$, with $d = 2, 3$ and is discretized using higher-order and highly-smooth spline spaces. This makes the matrix formation task very challenging from a computational point of view. We overcome this problem by introducing a low-rank decoupling of the operator into space and time components. Numerical experiments demonstrate the efficiency of this approach.

This work was done jointly with F. Scholz and I. Toulopoulos (RICAM - Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria).

6. Bilateral Contracts and Grants with Industry

6.1. Bilateral Contracts with Industry

● **NURBSFIX: Repairing the topology of a NURBS model in view of its approximation.** We have a research contract with the industrial partner GeometryFactory, in collaboration with the project-team Titane (Pierre Alliez). The post-doc of Xiao Xiao is funded by this research contract together with a PEPS from the labex AMIES.

Because of their flexibility and accuracy, NURBS (Non-Uniform Rational Basis Spline) models have become a standard in the modeling community for generating and representing complex shapes. They are made of several surface patches and a collection of curves that are used for trimming. As a direct consequence of software quirks, designer errors, and representation flaws, these NURBS models have inconsistencies that introduce small gaps and overlaps between surface patches. They are mainly located on the singularity graph of a NURBS model, near the trimming curves, especially near singularities such as sharp edges or corners. Building a correct approximation of a NURBS model in the presence of inconsistencies is a challenging problem. Most of the current approaches are based on the repairing of the geometry of the surface patches. This requires an interactive process which is difficult to control and rarely completely successful. In this project, we develop another approach which consists in repairing the topology of the singularity graph within a tolerance volume. This tolerance volume will be considered as a protected region that will not receive any query of geometric computations. Based on that, three types of approximations will be treated: triangular isotropic surface meshing of NURBS models, volume approximation of multi-domains delimited by NURBS surfaces, and NURBS models approximation within a given tolerance volume.

7. Partnerships and Cooperations

7.1. European Initiatives

7.1.1. FP7 & H2020 Projects

7.1.1.1. ARCADES

Program: Marie Skłodowska-Curie ETN

Project acronym: ARCADES

Project title: Algebraic Representations in Computer-Aided Design for complEx Shapes

Duration: January 2016 - December 2019

Coordinator: I.Z. Emiris (NKUA, Athens, Greece, and ATHENA Research Innovation Center)

Scientist-in-charge at Inria: L. Busé

Other partners: U. Barcelona (Spain), Inria Sophia Antipolis (France), J. Kepler University, Linz (Austria), SINTEF Institute, Oslo (Norway), U. Strathclyde, Glascow (UK), Technische U. Wien (Austria), Evolute GmBH, Vienna (Austria).

Webpage: http://arcades-network.eu/

Abstract: ARCADES aims at disrupting the traditional paradigm in Computer-Aided Design (CAD) by exploiting cutting-edge research in mathematics and algorithm design. Geometry is now a critical tool in a large number of key applications; somewhat surprisingly, however, several approaches of the CAD industry are outdated, and 3D geometry processing is becoming increasingly the weak link. This is alarming in sectors where CAD faces new challenges arising from fast point acquisition, big data, and mobile computing, but also in robotics, simulation, animation, fabrication and manufacturing, where CAD strives to address crucial societal and market needs. The challenge taken up by ARCADES is to invert the trend of CAD industry lagging behind mathematical breakthroughs and to build the next generation of CAD software based on strong foundations from algebraic geometry, differential geometry, scientific computing, and algorithm design. Our game-changing methods lead to real-time modelers for architectural geometry and visualisation, to isogeometric and design-through-analysis software for shape optimisation, and marine design and hydrodynamics, and to tools for motion design, robot kinematics, path planning, and control of machining tools.

7.1.1.2. POEMA

Program: Marie Skłodowska-Curie ITN

Project acronym: POEMA

Project title: Polynomial Optimization, Efficiency through Moments and Algebra

Duration: January 2019 - December 2022 (48 months)

Coordinator: B. Mourrain (Aromath, Inria Sophia Antipolis)

Other partners: LAAS - CNRS, Toulouse (France), Sorbonne Université, Paris (France), Centrum Wiskunde & Informatica, Amsterdam (The Netherlands), Stichting Katholieke Universiteit Brabant, Tilburd (The Netherlands), Universität Konstanz (Germany), Università degli Studi di Firenze (Italy), University of Birmingham (United Kingdom), Friedrich Alexander University Erlangen-Nuremberg (Germany), Universitet I Tromsø (Norway), ARTELYS SAS, Paris (France).

Webpage: http://poema-network.eu/

Abstract: Non-linear optimization problems are present in many real-life applications and in scientific areas such as operations research, control engineering, physics, information processing, economy, biology, etc. However, efficient computational procedures, that can provide the guaranteed global optimum, are lacking for them. The project will develop new polynomial optimization methods, combining moment relaxation procedures with computational algebraic tools to address this type of problems. Recent advances in mathematical programming have shown that the polynomial optimization problems can be approximated by sequences of Semi-Definite Programming problems. This approach provides a powerful way to compute global solutions of non-linear optimization problems and to guarantee the quality of computational results. On the other hand, advanced algebraic algorithms to compute all the solutions of polynomial systems, with efficient implementations for exact and approximate solutions, were developed in the past twenty years. The network combines the expertise of active European teams working in these two domains to address important challenges in polynomial optimization and to show the impact of this research on practical applications.

POEMA aims to train scientists at the interplay of algebra, geometry and computer science for polynomial optimization problems and to foster scientific and technological advances, stimulating interdisciplinary and intersectoriality knowledge exchange between algebraists, geometers, computer scientists and industrial actors facing real-life optimization problems.

7.1.1.3. GRAPES

Program: Marie Skłodowska-Curie ETN

Project acronym: GRAPES

Project title: Learning, Processing and Optimising Shapes

Duration: December 2019 - November 2023

Coordinator: I.Z. Emiris (NKUA, Athens, and ATHENA Research Center, Greece)

Scientist-in-charge at Inria: L. Busé

Other partners: U. Barcelona (Spain), Inria Sophia-Antipolis (France), J. Kepler University, Linz (Austria), SINTEF Institute, Oslo (Norway), U. Strathclyde, Glascow (UK), RWTH Aachen (Germany), U. Svizzera Italiana (Switzerland), U. Tor Vergata (Italy), Vilnius U. (Lithuania), Geometry-Factory SARL (France).

Webpage: http://grapes-network.eu/

Abstract: GRAPES aims at advancing the state of the art in Mathematics, Computer-Aided Design, and Machine Learning in order to promote game changing approaches for generating, optimising, and learning 3D shapes, along with a multisectoral training for young researchers. Recent advances in the above domains have solved numerous tasks concerning multimedia and 2D data. However, automation of 3D geometry processing and analysis lags severely behind, despite their importance in science, technology and everyday life, and the well-understood underlying mathematical principles. GRAPES spans the spectrum from Computational Mathematics, Numerical Analysis, and Algorithm Design, up to Geometric Modelling, Shape Optimisation, and Deep Learning. This allows the 15 PhD candidates to follow either a theoretical or an applied track and to gain knowledge from both research and innovation through a nexus of intersectoral secondments and Network-wide workshops. Horizontally, our results lead to open-source, prototype implementations, software integrated into commercial libraries as well as open benchmark datasets. These are indispensable for dissemination and training but also to promote innovation and technology transfer. Innovation relies on the active participation of SMEs, either as a beneficiary hosting an ESR or as associate partners hosting secondments. Concrete applications include simulation and fabrication, hydrodynamics and marine design, manufacturing and 3D printing, retrieval and mining, reconstruction and visualisation, urban planning and autonomous driving.

7.2. International Initiatives

7.2.1. Participation in Other International Programs

7.2.1.1. PHC Alliance

- Program: PHC Alliance

- Project title: High-order methods for computational design and data-driven engineering

- Duration: 01/2020–12/2021

- Coordinator: Angelos Mantzaflaris

- Other partners: Swansea University, UK

- Abstract: The aim of this project is to develop a mathematical framework for the integration of geometric modeling and simulation using spline-based finite elements of high degree of smoothness. High-order methods are known to provide a robust and efficient methodology to tackle complex challenges in multi-physics simulations, shape optimization, and the analysis of large-scale datasets arising in data-driven engineering and design. However, the analysis and design of high-order methods is a daunting task requiring a concurrent effort from diverse fields such as applied algebraic geometry, approximation theory and splines, topological data analysis, and computational mathematics. Our strategic vision is to create a research team combining a uniquely broad research expertise in these areas by establishing a link between the team AROMATH at Inria Sophia-Antipolis and Swansea University.

7.2.1.2. NSFC

- Program: NSFC

- Project title: "Research on theory and method of time-varying parameterization for dynamic isogeometric analysis",

- Duration: 2018-2021.

- Collaboration project with Gang Xu, Hangzhou Dianzi University, China.

7.3. International Research Visitors

7.3.1. *Visits of International Scientists*

Gang Xu, Hangzhou Dianzi University, China, visited AROMATH team (9 - 20 Oct.) to work on Isogeometric Analysis and Geometric Modeling.

Ibrahim Adamou, Univ. Dan Dicko Dankoulodo de Maradi, Niger, visited B. Mourrain (28 Oct. - 21 Dec.) to work on medial axes of curve arcs.

7.3.1.1. Internships

Martin Jalard (L3, Ecole normale supérieure de Rennes) for his *introduction to research* internship explored during 6 weeks (May 13th to June 21st) the application of Norton's lemma to the computation of isotypic decompositions.

7.3.2. *Visits to International Teams*

7.3.2.1. Research Stays Abroad

Evelyne Hubert was awarded a Simons fellowship within the program *Geometry, compatibility and structure preservation in computational differential equations*, from July to December 2019, at the Isaac Newton Institute in Cambridge (UK).

For the month of April, Evelyne Hubert was a guest professor a the University of the Arctic for *Pure Mathematics in Norway*.

Angelos Mantzaflaris visited in April the Computational Foundry, Swansea University, UK in the frame of the College of Science International Visitor Scheme.

8. Dissemination

8.1. Promoting Scientific Activities

8.1.1. Scientific Events: Organisation

8.1.1.1. General Chair, Scientific Chair

Laurent Busé organized a CIMPA school at Joao Pessoa, Brazil, November 4-13, on the topic "Syzygies: from theory to applications". Six courses delivered by international experts of this topic were scheduled during this school that hosted about 40 international PhD students and young researchers. For more details, see http://www-sop.inria.fr/members/Laurent.Buse/CimpaSchoolBrazil/.

8.1.1.2. Member of the Organizing Committees

Laurent Busé co-organized with C. D'Andrea and Y. Cid Ruiz a mini-symposium on «Syzygies and Applications to Geometry » at the SIAM Conference on Applied Algebraic Geometry, Bern, Switzerland, July 9-13, 2019.

Angelos Mantzaflaris and Bernard Mourrain co-organized with E. Tsigaridas a mini-symposium on "Symbolic-numeric methods for non-linear equations: Algorithms and applications" at the SIAM Conference on Applied Algebraic Geometry, Bern, Switzerland, July 9-13, 2019.

8.1.2. Scientific Events: Selection

8.1.2.1. Member of the Conference Program Committees

Laurent Busé was a member of the international program committee of the 2019 Symposium on Physical and Solid Modeling (SPM), Vancouver, Canada, June 17-19. He was also a member of the scientific committee fro the conference "Ideals, Varieties, Applications" celebrating the influence of David Cox, Amherst, USA, June 10-14.

Angelos Mantzaflaris served in the PC of the 2019 Solid and Physical Modeling (SPM), https://project.inria.fr/spm2019/.

8.1.2.2. Reviewer

Laurent Busé reviewed for *Symposium on Physical and Solid Modeling conference*, *SIGGRAPH conference*, *ISSAC conference* and *MEGA conference*.

Ioannis Emiris reviewed for the *Symposium of Computational Geometry*, *ICALP: Intern. Conf. Automata, Logic & Programming* and *MEGA: Méthodes Effectives en Géométrie Algébrique*.

Angelos Mantzaflaris reviewed for the *Symposium on Physical and Solid Modeling*.

Bernard Mourrain reviewed for *Symposium on Physical and Solid Modeling conference*, *ISSAC conference* and *MEGA conference*.

8.1.3. Journal

8.1.3.1. Member of the Editorial Boards

Ioannis Emiris is editorial board member of *J. Symbolic Computation (Elsevier)* and *Mathematics for Computer Science (Springer)*.

Evelyne Hubert is on the editorial board of Foundation of Computational Mathematics (since 2017) and the Journal of Symbolic Computation (since 2007).

Bernard Mourrain is associate editor of the Journal of Symbolic Computation (since 2007) and of the SIAM Journal on Applied Algebra and Geometry (since 2016).

8.1.3.2. Reviewer - Reviewing Activities

Laurent Busé reviewed for *Graphical Models journal*, *Advances in Applied Mathematics journal*, *SIAM SIAGA journal*, *International Mathematics Research Notices*, *Journal of Algebra*, *Applicable Algebra in Engineering, Communication and Computing journal*, *Computer-Aided Design journal*, *Computer Aided Geometric Design journal*, *Journal of Computer Science and Technology*, *AMS Mathematics of Computations journal*, *Israel Journal of Mathematics* and *Journal of Symbolic Computation*.

Evelyne Hubert reviewed for Journal für die reine und angewandte Mathematik, Proceedings of the American Mathematical Society, Advances in Applied Mathematics, Foundation of Computational Mathematics, SIAM Journal on Applied Algebra and Geometry, Journal of Symbolic Computation, and Computer Aided Geometric Design.

Bernard Mourrain reviewed for *Annales Henri Lebesgue*, *Collectanea Mathematica journal Computer-Aided Design journal*, *Computer Aided Geometric Design journal*, *Journal of Global Optimization*, *Journal of Symbolic Computation*, and *Mathematics of Computations journal*,

Angelos Mantzaflaris reviewed for Computer Methods in Applied Mechanics and Engineering, Computer Aided Geometric Design, Computers & Mathematics with Applications, and Journal of Symbolic Computation.

8.1.4. Invited Talks

Ahmed Blidia was an invited speaker in the minisymposium *Multivariate spline approximation and algebraic geometry* at SIAM Applied Algebraic Geometry meeting, Bern, Switzerland in June 2019.

Laurent Busé was invited to give two lectures at the "27th National School on Algebra: Graded modules over polynomial rings with applications to free divisors", Bucarest, Romania, May 19-25; he was invited to give a plenary talk at the conference "Ideals, Varieties, Applications", in honor of David Cox, Amherst, USA, June 10-14; he gave an invited course during the school "TIME2019: Curves and Surfaces, a History of Shapes", Levico Terme, Italy, Septembre 2-6.

Ioannis Emiris was an invited speaker at KAUST, Visual computing center, Saudi Arabia in February 2019, and SIAM Applied Algebraic Geometry meeting, Bern, Switzerland in June 2019.

Alvaro Fuentes Suarez gave a seminar talk in the MFX team at Inria NGE in February 2019.

Evelyne Hubert presented a series of lectures at the national conference *Equations Fonctionnelles et Interactions* (Anglet). She was invited to give talks at the A^3 - *Arctic Applied Algebra* conference (Tromsø, Norway, April 2019) and the conference *A celebration of Symmetry and Comptutation* (Canterburry, UK). She delivered seminar talks at the University of the Arctic (Tromsø, Norway) and at the Isaac Newton Institute (Cambridge, UK).

Angelos Mantzaflaris gave an invited talk at the 4th Workshop of the ERC project CHANGE, Centro Congressi dell'Annunziata, Sestri Levante, Italy (November 2019) and at the Schloss Dagstuhl – Leibniz Center for Informatics seminar on Interactive Design and Simulation, Germany (December 2019). He also delivered a mini-symposium presentation at the annual conference on Isogeometric Analysis (IGA 2019, Munich, Germany, September 2019).

Both Angelos Mantzaflaris and Bernard Mourrain were invited at the meeting on "Isogeometric Splines: Theory and Applications" of the Banff International Research Station for Mathematical Innovation and Discovery (BIRS, February 2019), at the Oberwolfach Mini-Workshop on Mathematical Foundations of Isogeometric Analysis (July 2019), and at the Algebraic Spline Geometry Meeting in Swansea, UK (August 2019).

Bernard Mourrain was invited to give a talk at the conference A^3 - *Arctic Applied Algebra* (Tromsø, Norway, April 1-4); an invited speaker of the conference MEGA (Madrid, Spain, June), to talk at the minisymposium *The algebra and geometry of tensors* of SIAM Applied Algebraic Geometry meeting, (Bern, Switzerland, June 2019), at the conference *Multivariate Approximation and Interpolation with Applications* (Vienna, Austria, Aug. 26-30), at the Italian Mathematical Union conference (Pavia, Sept. 2-5).

He was also invited to give two courses on splines at the University of Montpellier, (October 23-24, 2019).

Erick Rodriguez Bazan was an invited speaker in the minisymposium *Symmetry in algorithmic questions of real algebraic geometry* at SIAM Applied Algebraic Geometry meeting, Bern, Switzerland in June 2019.

Fatmanur Yildirim was an invited speaker in the minisymposium *Syzygies and applications to geometry* at SIAM Applied Algebraic Geometry meeting, Bern, Switzerland in June 2019.

8.1.5. *Leadership within the Scientific Community*

Ioannis Emiris is member of the Scientific Board of Hellenic Foundation for Research & Innovation (http://www.elidek.gr), representing Informatics and Mathematics.

Bernard Mourain has been elected vice chair of the SIAM Algebraic Geometry group.

8.1.6. *Scientific Expertise*

Evelyne Hubert was on the hiring committee for Junior Research Scientists at Inria NGE.

8.1.7. *Research Administration*

Bernard Mourrain is member of the BCEP (Bureau du Comité des Equipes Projet) of the center Inria- Sophia Antipolis.

Evelyne Hubert was a member of Inria Evaluation committee (until June 2019).

Laurent Busé is a member of the admnistrative and scientific committee of the labex AMIES. He is also member of the CDT at Inria Sophia-Antipolis and the CPRH of the Mathematics Laboratory Jean-Alexandre Dieudonné of the University of Nice.

8.2. Teaching - Supervision - Juries

8.2.1. *Teaching*

- Licence : Ioannis Emiris, Discrete Math, 52 h (L1), NKU Athens
- Licence : Ioannis Emiris, Software development, 26 h (L3), NKU Athens
- Master : Ioannis Emiris, Geometric data science, 52 h (M2), NKU Athens
- Master : Ioannis Emiris, Structural bioinformatics, 39 h (M2), NKU Athens
- Master : Laurent Busé, Geometric Modeling, 18h (M2), engineer school of the University of Nice Sophia-Antipolis (EPU).
- Undergraduate: Angelos Mantzaflaris "Fondements mathématiques 2 (L1 - TD Partie analyse), University of Côte d'Azur, spring semester 2019.

8.2.2. *Supervision*

- PhD in progress: Lorenzo Baldi, Structure of moment problems and applications to polynomial optimization. POEMA Marie Skłodowska-Curie ITN, started in October 2019, supervised by Bernard Mourrain.
- PhD in progress: Ahmed Blidia, New geometric models for the design and computation of complex shapes. ARCADES Marie Skłodowska-Curie ITN, started in September 2016, supervised by Bernard Mourrain.
- PhD in progress: Rima Khouja, Tensor decomposition, best approximations, algorithms and applications. Cotutelle Univ. Liban, started in November 2018, cosupervised by Houssam Khalil and Bernard Mourrain.
- PhD: Ioannis Psarros, Proximity problems for high-dimensional data. Greek scholarships foundation. Defended in Jun. 2019, NKUA, supervised by Ioannis Emiris.
- PhD in progress: Evangelos Bartzos, Algebraic elimination and Distance graphs. ARCADES Marie Skłodowska-Curie ITN, started in June 2016, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Clément Laroche, Algebraic representations of geometric objects. ARCADES Marie Skłodowska-Curie ITN, started in Nov. 2016, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Apostolos Chalkis, Sampling in high-dimensional convex regions, Google Summer of Code, started in June 2018, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Emmanouil Christoforou, Geometric approximation algorithms for clustering, Structural Bioinformatics national infrastructure "Inspire", started in Jan. 2018, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Tobias Metzlaff. Multivariate orthogonal polynomials and applications to global optimization. POEMA Marie Skłodowska-Curie ITN, started in December 2019, supervised by Evelyne Hubert.

- PhD in progress: Erick Rodriguez Bazan, Symmetry in Algebraic Computations. Inria. Started in November 2017, supervised by Evelyne Hubert.

- PhD : Alvaro Fuentes Suarez defended September 2019 [1]. Modeling shapes with skeletons: scaffolds & anisotropic convolution. ARCADES Marie Skłodowska-Curie ITN, started October 2016 and supervised by Evelyne Hubert.

- PhD in progress: Thomas Laporte, Towards a 4D model of the respiratory system. Fellowship from ED SFA/UCA. Started on October 2019, co-supervised by Benjamin Mouroy (UCA) and Angelos Mantzaflaris.

- PhD in progress: Riccardo Di Dio, Building a diagnosis tool to detect broncho-constrictions, BoostUrCAreer Marie Skłodowska-Curie COFUND fellowship. Started on November 2019, co-supervised by Benjamin Mouroy (UCA) and Angelos Mantzaflaris.

8.2.3. *Juries*

Bernard Mourrain was a member of the PhD committee of Matias R. Bender *Algorithms for sparse polynomial systems : Gröbner basis and resultants*, Sorbonne Université, Paris, June 3rd; a reviewer and member of the committee of the HDR of Frédéric Holweck entitled *On the projective geometry of entanglement and contextuality*, University Bourgogne Franche-Comté, Belfort, France, September 11th.

Evelyne Hubert was a reviewer for the Habilitation thesis of Georg Regensbuger, Johannes Kepler University (Austria): *Algebraic and algorithmic Approached to Analysis: Integro-differential equations, positive steady states, and wavelets*

Laurent Busé was a reviewer and member of the committee the PhD thesis of Navid Nemati, *Syzygies: Algebra, Combinatorics and Geometry*, Sorbonne Université, Paris, May 28; he was also a member of the PhD committee of Matias R. Bender, *Algorithms for sparse polynomial systems : Gröbner basis and resultants*, Sorbonne Université, Paris, June 3rd.

8.3. Popularization

8.3.1. *Interventions*

Ioannis Emiris was an invited speaker at "From Open Access to Science", Athens, May 2019. and "30 years celebration of ATHENA Research Center", Athens, November 2019.

9. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

[1] A. J. FUENTES SUÁREZ. *Modeling shapes with skeletons: scaffolds & anisotropic convolution*, Université Côte D'Azur, September 2019, https://hal.inria.fr/tel-02420728

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